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COMPLEMENTARY DIGITAL PRODUCTS AND DIGITAL PIRACY

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Preface

The following Ph.D. thesis is the result of these three years of study and efforts. During the construction of the model we faced various difficulties, but we always proceeded to develop and hopefully increase the quality of this work. This manuscript is the second version of this thesis, and has been extensively redacted from the first version presented in October 2019 under the counsel of the gentle reviewers.

In detail, under the revisions suggested by prof. Dimitri Paolini, we made the following integrations: we introduced a full Chapter on literature reviews, now defined as Chapter 1. We also dedicated a Section to the framework on which our model is based, the Mussa and Rosen (1975) approach. In Chapter 2 and in Chapter 3 we modified the related literature section, to allow for a better and clearer references to the relevant literature. Also, in both chapters we clarified both the assumptions and the results presented in our models. Last but not least, in both Chapter 2 and 3 we further developed both our results analysis and changed the exposition of our conclusions.

According to the suggestions by prof. Bernardino Cesi, we further developed the exposition of our results and we clarified the assumptions of the models with focus on the literature basis for our choices. Under his suggestions, we also proposed a deep analysis of the role of the fine on the model, with a clearer explicitation of the related results. In preparation for publication, we also developed our result analysis to include more references to empirical works in the field. Although we were not able to introduce a welfare analysis in this thesis, we instead enhanced our exposition

with respect to the previous version of the thesis on the comparison between the benchmark model and the indirect appropriation model with and without network effects, which leads to stronger results and in our opinion to interesting insights.

We wish to express our full gratitude to both reviewers, Prof. Dimitri Paolini and Prof. Bernardino Cesi for the time dedicated to reviewing our work and for their constructive and helpfull remarks.

In the following manuscript, in Chapter 1 we will introduce a comprehensive review of literature on digital piracy, with a focus on the theoretical literature with a Mussa and Rosen (1975) approach to vertical differentiation. In Chapter 2 we will introduce a model for indirect appropriation across complementary information products, evaluating the underlying mechanics of complementarity between goods and then studying the effects of digital piracy on those mechanics. In Chapter 3 we extend the previous model to accomodate network effects, where network effects are modelled as a positive within-effect network externalities due to consumption.

Chapter 1

Information goods and Digital piracy: a literature review

1.1 Introduction

The age of the internet has brought us into frequent contact with the reality of Digital Technologies. This contact leads to significant changes in today's economic activities on various levels and, for economists, it has become necessary to understand how standard economic models change in the new environment[1].

Thanks to digitalization, information can be encoded in form of zeroes and ones[2] but, to be shared, information needs to be further encoded and transformed into Information Goods and, if the format is digital, the Information Good becomes a Digital Good [3]. Digitalization also has a great effect on various production and economic costs, such as search costs, replication costs, transportation costs, tracking costs and verification costs [4].

[1] Goldfarb and Tucker (2019)
[2] Shapiro and Varian (1998)
[3] Belleflamme (2016)
[4] See Goldfarb and Tucker (2019) for a deeper analysis of literature on how such costs affect the digital market

Shapiro and Varian in their 1998 work defined the Internet as a "giant, out of control copying machine". This definition is still fitting and today reflects a fully fledged reality. But the reasons copying is so easy, is the nature of Information goods: Information Goods are, in fact, inherently non-rival and non-excludable.

Information Goods are non-rival as the consumption of one good by one consumer does not reduce the quantity or quality of goods available to other consumers. This implies that *"at any level of production of the good, the marginal cost of delivering it to an extra consumer is close to zero"*[5]. Also, Information Goods are non-excludable unless there is a legal or technological effort. Thus, no one can exclude another individual from consuming the information normally. This means that firms are not able to appropriate the revenues due to their innovative effort. As the innovative effort may imply consistent sunken and fixed costs, in the long term there will be less incentives to create. This problem is commonly known in literature as *under-production*[6].

To tackle the problem of under-production, Intellectual Property rights grant the creator of a certain Information Good the exclusive use and distribution of the Information Good itself. Although this solution allows creators to appropriate their revenues, it also leads to a positive price when the marginal cost of production is zero and consequently to a loss in welfare. This problem is known in literature as *under-utilization*.

To achieve balance between the problem of under-production and under-utilization, Intellectual Property right grants exclusive rights only for limited period of time and for limited scope[7], based on the type of Information Good we are facing.

Legislation on Intellectual Property rights worldwide tend to divide different Intellectual Property types based on the subject of the Information Good: for inventions, processes and industrial design the Intellectual Property regimes is defined

[5] Belleflamme(2016)

[6] on this topic we will follow the lines set in Belleflamme and Peitz (2010), Belleflamme and Peitz (2014) and Belleflamme (2016)

[7] Belleflamme and Peitz (2010)

by *patents* and/or *trade secrets*. On the other hand, brand names are covered by *trademarks*. Last but not least, *Copyright* treatment encompasses all works which involve the expression of an author's creativity, in all its forms, genres, media and styles. This includes all sorts of art industries such as the music industry, the motion picture industry, the literature industry, theatrical productions, performing arts, visual arts but also works such as architectural designs and photographs. The main difference between patent, trademark, trade secrets and copyright is that the first three protect industrial property, while copyright protects art.

The term Copyright itself derives from the 1710's Statute of Anne as *"An Act for the Encouragement of Learning, by Vesting the Copies of Printed Books in the Authors or Purchasers of such copies, during the Times therein mentioned"*[8] and granted the exclusive right to produce copies of books and other writings. Copyright originally was set with a term of 14 years, with a possible extension to further 14 years for a total of 28 years both in the UK with the 1710 Statute of Anne and in the US with the Copyright Act of 1790. Today most countries in the world adhere to either or both the Berne Convention and the Agreement on Trade Related Aspects of Intellectual Property Rights (TRIPS), although certain countries such as the Marshall Islands do not grant any Copyright Protection and are not signatories of any treaties on Copyright. Note that both the Berne Convention and TRIPS grant a minimum copyright length equal to the life of the author of the work plus 50 years. Signatories of both treaties may grant longer terms than the one established in the Berne Convention and Trips. The US and EU for example grant a length of the life of the author of the work plus 70 years, while Mexico since 2003 has granted a length of copyright equal to the life of the author of the work plus 100 years.

It is also interesting to note that in recent years, the increase in length of the Copyright terms in most nations is due to intensive lobbying of legislators by big IP companies. This lobbying process has been documented in numerous works such as

[8] The statute of Anne, 1710

Ota (1998), Benerjee (2006) and Schlackman (2018).

The main reasons firms give to justify their intensive lobbying is the phenomenon known as *Digital Piracy*.

In the following Chapter, we will first define in the next section what digital piracy is. We will then proceed to understand who practice it and why. In Subsection 4 we will proceed to lay the foundation of the analysis on digital piracy and the main results, with focus on the Mussa and Rosen theoretical framework. From these foundations in Subsection 5,6 and 7 we will then introduce the three main positive effects of digital piracy known in academic literature, while in section 8 we will evaluate the most recent frontier of economic analysis on digitalization and digital piracy. We will conclude in Subsection 9 with some concluding remarks.

1.2 What is Digital Piracy?

The main question is now to determine what Digital Piracy is. In Belleflamme and Peitz 2014, the authors define digital piracy as follow: *"Digital Piracy is the act of reproducing, using or distributing information products in digital formats and/or using digital technologies, without the authorization of their legal owners, where information products are a type of intellectual property (IP) products such as books, music, movies or paintings"*. In Marshall (2004) instead, the definition is: *"Digital piracy is [...] a wide variety of activities including counterfeiting, pirating, bootlegging, home taping, tape trading and online file sharing."*. Although the definition by Marshall may be extremely fitting for peer-to-peer platforms, it is also narrower than the one offered by Belleflamme and Peitz (2014), thus in this work we will follow the definition presented in Belleflamme and Peitz (2014) as it is simple and effective.

Digital piracy is a very strong and pervasive phenomenon across countries and sectors and it is a multi-billion-dollar industry that continues to grow. The Frontier

Economics' 2017 Economic Impacts of Counterfeiting and Piracy Report for BASCAP[9] and INTA[10] shows a value of digital piracy in movies, music and software equal to 213 billion USD in 2013, of which 160 billion USD in digital piracy in film, 29 billion USD in digital piracy in music and 24 billion in digital piracy in software. The forecast for 2022 shows values between 384 and 856 billion USD. Of this amount, a share between 289 and 644 billion USD is estimated in the motion picture industry, a share between 53 and 117 billion USD in the music industry and a quota between 42 and 95 billion USD in the software industry. The cost of both digital piracy and counterfeiting both in economic and social terms has been estimated in 2013 between 737 and 898 billion USD, while the projection shows values ranging from 1.54 to 1.87 trillion USD in 2022. Similar results have also been shown in 2016 UK intellectual property office report. Poort and Weda (2015) also showed how piracy rates are still growing in the industry notwithstanding all the efforts of firms in the affected industries to avoid piracy.

To tackle this phenomenon, firms have set up organizations such as the Alliance for Creativity and Entertainment, the Intellectual Property Owners association and the Recording Industry Association of America, while law enforcement agencies all over the world are sparing no effort trying to reduce digital piracy through DNS blocking and crackdown on online illegal distribution platforms, but as of todaythose actions appear to have had marginal if any effect on the rate of digital piracy[11].

We also have to consider that Digital Piracy has evolved at quite a fast pace. In 1999 Napster was the first peer-to-peer (P2P) file sharing community. Even after the music industry won its legal procedings and was allowed to shut down Napster in 2001 and subsequently the platform Gorkster in 2005, Digital Piracy did not

[9]The acronym BASCAP stands for "Business Action to Stop Counterfeiting and Piracy". BASCAP is an Internationa Chamber of Commerce initiative that connects various companies from business across industries, sectors and national borders to strenghten each member's ability to protect their brand and investment from counterfeiting and piracy.

[10]INTA. is the acronym of the International Trademark Association, a global non-profit advocacy associacion of brand owners which supports brand owners in their effort to protect trademarks and related Intellectual properties.

[11]Poort and Weda (2015)

show any decrease. In fact, the technology for file sharing continued to evolve with the creation of applications such as E-Mule and Bit-Torrent. Such softwares are operated on decentralized networks, thus making shutdown impossible while at the same time granting access to digital piracy to even the most average internet users. It has to be said that both legal and technological interventions against digital piracy have shown scarce if any results in regards to their ability to lower digital piracy rates in the market. The results are so poor that Dootson, Pappalardo and Suzor (2016) write: *"Instead of investing resources into legal proceedings, we suggest that rights-holders should invest in innovative platforms that provide consumers with greater access to content in a timely manner at a fair price."*[12]. The doubts on the effects of legal and technological intervention against digital piracy concern not only the level of digital piracy in the market, but also the ability of firms to appropriate back revenues. In her 2017 study, McKenzie showed that there is no significant proof that legal intervention against digital piracy has any effect on the box office revenues for new movies[13].

To better understand the phenomenon, we have to delve deeper in understanding who is using digital piracy and why they are doing so.

1.3 Digital Piracy: Who and Why?

One of the most widely discussed topics in academic literature regards the characteristics of digital pirates and their motives.

On one hand, there are some organizations with a clear criminal intent of organizing a large scale reproduction and distribution of copyrighted material with the intent of earning profits. This type of act is defined in literature[14] *for-profit piracy* and is also known as *commercial piracy*. This type of piracy is similar to counterfeiting for

[12]Dootson, Pappalardo and Suzor (2016)
[13]McKenzie (2017)
[14]Belleflamme and Peitz (2010), Belleflamme and Peitz (2014)

physical goods and it is strictly connected to economic literature in that field. This branch of literature is not the object of our review, although recent works such as Tunca and Wu (2013) show that action against commercial piracy may have a role in affecting individual piracy.

End-User piracy (also known as *individual piracy*) is defined as the "illegal reproduction of copyrighted work by the consumers themselves"[15]. The "who" in this case is not a criminal organization, but a normal citizen that would otherwise be law-abiding.

In End-User piracy the reasons behind the decision to recour or not to digital piracy has been quite debated in literature. The puzzling problem has been tackled numerous times by various authors. Between the main factors that may influence such decisions we have the age of the consumer, his income and wealth, legal settings in the country and even cultural habits.

On this topic, Hill (2007) underlines three main families of reasons from which digital piracy arises and which can encompass various literature works on this topic.

The first family of causes is connected to the moral development stage of the individual and on how this development interacts with the external factors such as social norms and national culture. In psychology, Kholberg (1969) distinguishes three levels of moral development: pre-conventional level, conventional level and post-conventional level.

In the pre-conventional level the individual has a certain moral development with internal focus and with the objective of obtaining self-gratification while avoiding punishment. This is the level for children and morally immature adults. In the conventional level the individual feels the peer pressure and thus choices depend also on the disapproval by others. In this level the influence of social norms and laws is the main moral compass. This is the level of normal adults. Last, we have the postconventional moral level where the individual is concerned with the ramifi-

[15]Belleflamme and Peitz (2010)

cation of his action toward the society in general. Normally this equates to strong ethical principles for the individual.

Given these settings, Hill argues how digital piracy is practiced in a disproportional way by young and by males[16], which have a pre-conventional moral level, which is enhanced by the lack of a perceived risk of being detected and punished. Not only that, but he also argues how the lack of strong social norms and punishment against digital piracy also does affect individuals in the pre-conventional phase but also in the conventional phase, due to the lack of social peer pressure on the individual. This type of motivation is coherent with numerous works, such as Liebowitz (2004), Glass and Wood (1996) and are extended on national culture influence on digital piracy in Kini, Ramakrishna and Vijayaraman (2004).

The second family of causes for digital piracy argued by Hill is embedded in the equity theory, where *"Equity theory describes an individual's search for fairness and equity in social exchanges [...] Equity theory predicts that when individuals find themselves participating in relationships that are perceived to be inequitable, they will become distressed and will try to eliminate the distress by taking actions to restore equity."*[17]. In this case the digital pirates claim the moral high ground advocating that prices of digital goods are too high, thus they advocate that their illegal behaviour is motivated by their feelings of inequity.

The third cause for digital piracy argued by Hill is the moral intensity theory toward unethical actions. This theory is based on Jones (1991) in which the author considers a multi-dimensional construct with six dimensions. The six dimensions are: the magnitude of consequences in term of the harm done to the victim of the unethical behaviour, the social consensus toward the unethical act (social norms), the probability that the act will harm others, the length of time between the action and the harmful consequences, the closeness of the individual in term of social,

[16]This is supported in numerous works. The author cites D'Astous, Colbert and Montpetit (2005), Glass and Wood (1996) which are cited by Hill, but the results are also coherent with works such as Liebowitz (2010) and Cox, Collins and Drinkwater (2010)

[17]Hill (2007)

cultural, psychological or physical between the individual that makes the unethical action and those harmed by the act and last but not least the concentration effect, which corresponds to the number of people affected by the unethical act.

While Jones argues that those constructs are connected and show how individuals have no moral absolutes but are strongly influenced by context, Hill applies this theory to digital goods as *"the moral intensity is rather low, [...] there is not a strong social consensus that digital piracy is unethical. [...] The probability that copying a digital good will do harm is low [...] from the perspective of the copier. [...] Temporal immediacy is perceived to be irrelevant given that piracy is seen as doing little harm [...], Internet [...] act as a buffer between the pirate and the copyright holder, creating perceived distance and depersonalizing the crime. [...] The impact of piracy is seen to harm an institutional entity, such as a corporation, rather than individuals, the concentration effect is rather low."*[18] These results are also coherent with what shown in Cho, Chung and Filippova (2015) on the influence of social norms on individuals.

In conclusion, we can say that consumers recour to digital piracy for a complex plethora of reasons: some try it because they feel unfair the price of some digital goods and thus are not willing to pay the price and see digital piracy as a free substitute for the original digital good, some recour to digital piracy because they want immediata access to digital content that is not yet available in their geographical region, others recour to piracy because a lack of complex psychological constraint due to lack of personal morality or lack of social pressure or even for a complex set of moral values. Either way, digital piracy is widespread worldwide and strongly affects the digital market for digital goods with a worldwide sales displacement estimated in 2017's report for BASCAP and INTA on the Economic Impacts of Counterfeiting and Piracy for year 2020 between 737 and 898 billion USD.

Moving forward in our analysis, we can now make an in depth analysis of the eco-

[18] from Hill (2007). Those conclusions, as shown in Hill (2007), are coherent with the works of Kini et al (2004); Tan (2002) and Logsdon, Thompson and Reid (1994)

nomic literature on digital piracy. One of the main classification in the field derives from Peitz and Waelbroeck critical review of theoretical literature published in 2006 in the Information Economics and Policy Journal. In their review Peitz and Waelbroek identify four main branches of theoretical papers based on the arguments made by various authors: papers on the basic analysis of end-user copying, papers on indirect appropriation, papers on network effects and papers on consumer information and sampling effect. To those four existing categories we can add a fifth: in the last decade economic literature on digital piracy has evolved to encompass studies on the new business models which arise to tackle digital piracy. In the following subsections, we will analyze the five main branches of economic literature and define the theoretical and empirical framework existing in each branch of literature.

1.4 Basic analysis of Digital Piracy

Basic analysis of digital piracy is the first branch of literature we cover in this review, as it sets the basic information and assumptions underpinning the current copyright laws and the common principles underlying digital piracy and its effects on markets. In this field of economic research, the common approach of the various relevant papers is to study pricing information of a digital good in the presence of end-user digital piracy on the market.

The common methodology is to consider a theoretical framework based on product differentiation, given by the fact that the original has a certain quality q for $q \in [0, 1]$, while the unauthorized copy will have a lower quality $q^c \leq q$,[19] and the main objective is to explain the effect of digital piracy on the market.

In this field one of the main contributions is Novos and Waldman (1984). Novos and Waldman set two possible strategies for consumers: they can either buy the original good or they can copy the good with a certain cost. This cost is given by

[19] Peiz and Waelbroek (2006), Belleflamme (2016)

the marginal cost of production for the good c, plus an additional marginal cost of making a copy which is heterogeneous for consumers and represent their ability to make copies and a certain level of copyright protection[20]. In this setting, if the firm offers a sufficiently low price copying becomes unattractive and consumers will prefer the original. Otherwise, some consumers will choose to consume the original good, some consumers will choose to consume the copy. Although this model was originally drawn up for physical copies of a product, it fits consumers heterogeneity in the required knowledge necessary for bypassing software protection on digital products perfectly. The results of the study show how as long as the density of the heterogeneous cost z is increasing, an increase in copyright protection leads to an increase in the quality offered by the monopolist and thus some consumers will switch from copying to buying the original. Thus, ceteris paribus, stricter copyright protection does not lead to under utilization and consequently a stricter copyright enforcement in the market is consequently preferable. In conclusion, Novos and Waldman show that as copying involves higher social marginal costs, an increase in copyright protection may increase social welfare due to the shift of consumers from making copies to purchasing originals.

Those results fit well with the theory of Arrow (1962) and show how digital piracy results in less profits for the Copyright Owners of original works in the short run, which consequently generate the classic trade off between less investment in the production of original works in the long run causing the problem known in literature as under-production of digital products and the problem of under-utilization which derives from the creation of a legal monopoly.

Various subsequent models in this branch of literature confirmed those results, reaching the general consensus that although the presence of non-authorized copies limit the monopolistic power of the monopolist, thus generating welfare surplus in

[20]The total cost is defined as $c + z(1 + x)$, where x is the strenght of copyright protection, c is the marginal cost of reproduction and z is an additional marginal cost to make an additional copy of the original.

the short-run, in the long run it will lead to a loss in the incentives to produce new digital products. Noteworthy models are that of Bae and Choi (2006), in which the authors use a Mussa-Rosen approach for vertical product differentiation [21] and the model by Johnson (1985) which showed how the presence of digital piracy leads to a loss of variety in originals and in lower incentive to produce new originals.

1.4.1 The Mussa-Rosen theoretical framework

In this Subsection we will focus on why the Mussa-Rosen approach is important and widespread in this field of economics.

The Mussa-Rosen setting[22] is one of the main approach in theoretical literature in the basic analysis of end-user digital piracy and has been applied in various relevant contributions, such as Belleflamme (2003), Yoon (2002) and the aforementioned Bae and Choi (2006). The most common approach is to consider a monopolist which faces a heterogeneous group of consumers. Consumers heterogeneity, generally indicated by θ, may be discrete or continuous in an interval [0,1]. In the first case we will distinguish between high-valuation consumers and low-valuation, i.e. given a certain valuation threshold θ', consumers with taste parameter $\theta > \theta'$ are high valuation, otherwise are low-valuation consumers. Normally, in this case, a high valuation consumer will prefer the original over the copy in all cases, while consumers with low valuation may choose the copy over the original. In the second case the distribution is typically assumed to be uniform on the interval $\theta \in [0, 1]$. Various works also consider a certain relevant cost of copying the original that consumers may have to sustain to copy c and a cost to allow for the bypass of copyright protection x[23].

[21] (Mussa and Rosen (1978)

[22] It is important to Notice that when referring to the Mussa-Rosen setting or framework, we will always present a notation which stems from Peitz and Waelbroek (2006) and Belleflamme (2010).

[23] more information on this in in the previous Section when illustrating Novos and Waldman (1984) contribution.

The discrete approach is used i.e. in Cremer and Pestieau (2005). In this paper the authors consider a population of consumers with valuation j, with $j = 1$ if they have high-valuation for the good or with $j = 2$ if they have low-valuation for the good. The firm does not know the consumer type j. Given this setting, the firm offers a price-quality pair of goods (a good with low quality q_L at price p_L and a good with high quality q_H at a price p_H). Consumers derive different utilities from different quality good for consuming the original good, while they will have to pay the social cost x if they choose to copy, which is increasing based on the quality of the copy (under the constraint that the quality of the copy must be inferior of the quality of the original $q_c < q_H$) and the effort e to avoid the level of copyright protection. Given these settings, consumers will derive utility $u_j(q_i) - p_b$ for $i = L, H$[24], while the utility derived from copying is given by the maximum quality consumers with low-valuation may extract from the copy, net of the cost for copying and the effort needed to bypass the level of copyright protection $\max_q u_2(q) - x(q, e)$[25]. The results show that the social optimum when the government control the pricing strategy would be for the firm to sell only to high valuation consumers while allowing low valuation consumer to copy the original product, while instead for the firm it is optimal to sell to both types of consumers while implementing high effort e in copyright protection. Furthermore, the authors allow the effort e to be considered as the level of IP protection offered by the government. In this case the social optimum when the government controls the pricing strategy is to not provide any copyright protection e.

Another work that uses the discrete approach to the Mussa-Rosen setting is Takeyama (1997), in which the author extends the digital piracy model in a dynamic setting with two periods. In his work, Takeyama shows that although the negative effects of digital piracy are enhanced in a dynamic setting, it may be possible for the firm to extract the full consumer surplus from high-type consumers through a binding

[24] Notation from Peitz and Waelbroek (2006)

[25] notation taken from Peitz and Waelbroek (2006)

participating constraint in the first period.

The continuous approach to the Mussa and Rosen setting is instead used in Belleflamme (2003), Yoon (2002) and Bae and Choi (2006). Consumers while consuming the original good a consumer will derive a utility $\theta q - p$, where $\theta \in [0, 1]$ is the consumer taste valuation for the quality of the good, q is the quality of the original and p is the price of the original good, while for the copy consumers will derive a utility $\alpha\theta q - c$ where $\alpha \in [0, 1]$ is a quality depreciation of the copy w.r.t. the original and c is the cost of copying the goods. Given this setting, the monopolist profits will decrease given the availability of the copies in the market: copies may be seen as an inefficient Alternative Good of lower quality (as $\alpha \in [0, 1]$, it holds $\alpha\theta q < \theta q$ $\forall\theta \in [0, 1]$ and $q > 0$), which is produced at cost c. Belleflamme and Picard (2004) also extend this setting in a multi-product scenario. The results in this case show that while the demands for each product are independent normally, as the technology that allows for copying has increasing return to scale, whenever copies are in the market the demand for the various products becomes interdependent. An example was in the cd/dvd era, the cost to buy a program that allowed for cracking the IP protection on originals allowed to copy various movies. The results show that when setting a sufficiently low price they can avoid copying, although whenever there is only one firm digital piracy becomes unavoidable.

Also in continuous setting, the contribution from Alvisi, Argentesi and Carbonara (2002) extends the continuous Mussa-Rosen framework. In their work they analyze if and why a firm should offer a downgraded version of their products when in the market there is digital piracy. In their work, copies are assumed to have the same quality as originals, and to buy the original the price of the original has to be set lower than the cost of copying sustained by the consumer. The authors then introduce consumers with low or high copying costs. Given these settings, low-value consumers will be attracted by a low-quality version of the product, while high value consumers will be attracted by the high-quality product. The optimal

strategy for the firm in absence of digital piracy is to offer a single product, while when digital piracy is introduced in the market the optimal strategy is to introduce a low-quality version of the digital product: if the firm set the price lower than the copying cost for each consumer class, consumers will not have an incentive to copy. They also argue that in the presence of stricter enforcement of copyright, firms will have less incentives to differentiate their products based on quality, thus suggesting that vertical differentiation is the natural market reaction to digital piracy.

A similar issue has been studied in Cho and Ahn (2010). In their work, the authors use a somewhat similar setting to the one shown in Alvisi et al (2002): in both settings the quality between the original and the copy is the same and the cost of copying is the same for all consumers. In their setting Cho and Ahn use a discrete Mussa-Rosen approach, instead of the continuous approach seen in Alvisi et al (2002), with two types of consumers: high valuation consumers and low valuation consumers. Given this setting, one main difference between the two contributions is that even when Digital Piracy is absent from the market the firm will produce different quality versions of the digital product, targeted to the different consumer types (a high-end product targeted to the high-valuation consumer and a low-end product targeted to a low-end consumer). The results show that although it is true that digital piracy in the model leads to the production of digital goods of low quality, leading to inefficiencies, it is also true that such effect affects only the high-end version of the product and not the low-end version, in a somewhat similar way to what seen in Cremer and Pestieau (2005). In conclusion, Cho and Ahn (2010) also reach the consensus that strengthening in copyright protection may alleviate the problem.

It is important to understand this setting as it has been in numerous analyses also for models with network effects and of consumer information and sampling, thus it is a solid theoretical framework which spans over multiple branches of digital piracy literature.

1.4.2 Highlights on the basic analysis of digital piracy

As we saw in Section 1.4, the basic analysis of digital piracy is quite united in its findings about digital piracy and its effects on the market: to reiterate, although digital piracy may lead to higher welfare in the short run due to the lessening in the monopolistic power of the copyright owner, in the long run digital piracy leads to a lack of incentives to produce high quality goods on certain level, thus it would be advisable to strengthen copyright enforcement to lessen the effects of digital piracy on the market.

Although it may appear that this branch of literature is quite united in its results, in recent years contributions such as Waldfogel (2012) and of Aguiar and Waldfogel (2016) seem to suggest a less clear outcome on the effects of digital piracy on the market. In Waldfogel (2012) the author assesses the quality of the new recorded music since the advent of peer-to-peer sharing in the form of Napster through three independent approaches. The three approaches are based respectively on an index of the quantity of high-quality music based on critics' retrospective lists, on a comparison in the music sales and last but not least on data about the airplay. The analysis showed no evidence of a reduction in the quality of music, thus the author suggests that *"Researchers and policymakers thinking about the strength of copyright protection should supplement their attention to producer surplus with concern for consumer surplus as well"*. Aguiar and Waldfogel (2016) extended the work of Waldfogel (2012), showing that music quality is increasing worldwide: on one hand music quality is growing thanks to artists on independent labels or new artists and that despite the growth in number of products, sales are getting more and more concentrated.

While it is still true that Digital Piracy is an illegal and condamnable behaviour, various works in recent years have shown that digital piracy in certain situations may be profitable both in the short and in the long-run, making the border between damage and profit even less clear. The three main mechanisms that are studied in

literature on the *profitable piracy* current are indirect appropriation, network effects and consumer sampling which will be examined in the following sections.

1.5 Indirect Appropriation

One of the earliest and main arguments in the positive piracy branch of economic literature is the process of *indirect appropriation*. Indirect appropriation arises in a market where illegal copies are produced whereas the copyright owner is able to capture part or all the value lost because of said copies. The effect of indirect appropriation has been observed in two main cases. In the first case, the market registered an increase in the users' willingness to pay for the original goods whereas in presence of a copy. In this case, we face the so-called *indirect appropriation through consumers*[26]. In the second case, a given users' willingness to pay for the original good increases whereas in the market a complementary good is present. In this case we face *indirect appropriation across products*[27].

Indirect appropriation across consumers

As aforementioned, indirect appropriation across consumers happens whenever the market registers an increase in the users' willingness to pay for the original good whereas in presence of a copy. The first work in this category is also one of the oldest in this field: Liebowitz (1981 and 1985). The first work of Liebowitz in 1981 focused on the effect of reprography on the copyright system. In this work, Liebowitz showed that the effect of copying depends on the relative sizes of the markets for originals and copies, showing that price discrimination may be a better solution to the problem posed by photocopying machines becoming widespread. In his 1985 work, Liebowitz evaluated the empirical impact of photocopying on publishers of economic journals. He argues that *in order to retain the same degree*

[26]Peitz and Waelbroek(2006)
[27]Peitz and Waelbroek(2006)

of appropriateness [...] the publisher would need to be able to price discriminate among purchasers of originals, charging a higher price for those originals that would be used to make many copies.

Liebowitz compared data from institutional and individual subscription prices for 80 economic journals, the number of citations received by each journal, the age of the journal and the type of publisher. The results showed that there is a positive relation between usage and the relative prices of such journals, that there is an increase in price discrimination after the introduction of new photocopying machines. Also, the results show how the demand for material ,which is easier to copy, increases based on how easy it is to copy the original product. This last result is quite important, as it allows the author to argue how indirect appropriation may be one of the drivers in the major shift of academic literature from books to journals: as books are weak complement to photocopiers, while journals are strong completement to photocopiers, the rise in demand for journals caused by indirect appropriation leads to the shift between media. The results from those works may be simplified in saying that the effect of digital piracy on the market is not clear, as it depends on different variables such as the size of the degree of substitutability perceived by consumers during consumption between originals and copies, the maximum number of copies made by each original and the relative size of the market for originals and copies when in the presence of operating costs in the markets for copies and originals, while whereas there is no costs associated with the markets total welfare, consumer surplus and producer profits are always increasing as a result of copying.

In Besen and Kirby (1989) the authors present a general model for direct and indirect appropriability, focusing on the extent to which originals and copies are regarded as substitutes by consumers and the "technology" for making copies from originals. To do so, they present three variations of the original model: in the first case, the marginal cost of copying is constant but greater than the marginal cost of producing originals, in the second case originals and copies are perfect substitutes

but the marginal cost of copying is increasing and in case three originals and copies are imperfect substitutes and the marginal cost of copying is increasing. In the first case, the copyright owner is forced to lower his price to compete with copies, leading to a reduction in the demand for the originals due to the existence of copies and consequently this case leads to a reduction in the producer profits. On the other hands, consumers will gain either because the price of originals is reduced or because they can consume a copy. In both the second and the third case, independently from the substitutability of original and copies, the copyright owner will be able to enact indirect appropriation. In the second scenario: the price of original rises as the original is shared amongst multiple users. In conclusion we can say that independently from the substitutability of original and copies, when cheaper copies are substituted with more expensive originals, both consumers and copyright owners are better off, while when copies and originals have both low production cost, then producers will lose and consumers will gain.

The contributions from Liebowitz and Besen and Kirby set the starting point for an important principle in the indirect appropriation across consumer field: the aggregation of consumers in clubs (or libraries in the case of Liebowitz studies) allows copyright owner to price-discriminate and thus to indirect appropriate back part or full profits.

This mechanism has been extended and specified in Bakos et al. (1999): in their work the authors highlight two main effects of sharing as the *aggregation effect* and the *club diversity* effect.

Under the conditions of end-user piracy (presence of a copyright owner with market power, the shared good being a non-rival good and the social norms that sharing is not part of an efficient mark for second-hand goods). Under the assumption that the marginal cost for additional copies of the Information Good is zero for the seller, the consumer valuation for the Information Goods are independent and uniformly distributed, that the shared copies are perfect substitutes to the originals

and can be made costlessy within teams and that the teams are of uniform size, then the larger the size of the team sharing the product, the greater will be the profits for the copyright owners. The authors then relax the previous assumption, showing that the profits will increase for the copyright owner even when sharing is not efficient from a technological point of view. This effect due to "bundling" consumers together is called *aggregation effect*. When the authors further relax the assumptions, introducing a positive correlation between club members and relaxing the assumption on the club being of the same size, the distribution of the club valuations will flatten, thus leading to lower profits. In conclusion, the net effect of sharing is given by the balance between the aggregation and the club effects.

Although those works still stand as important contributions, under the changes imposed by the digitalization it seems that those mechanisms are now non operative. In both Watt (2005) and Johnson and Waldman (2005), the authors reflect on how digitalization leads to an overwhelming flooding in the market for illegal copies. Johnson and Waldman define flooding as the situation whereas *"there are more potential copies for sale than there are potential buyers of the copies, so the price of a copy falls to the marginal cost of producing a copy"*, thus leading to a profound change in the analysis presented in previous papers on indirect appropriation: *when copies flood the market, the price of a copy falls to marginal cost and thus there is no profit associated with selling copies and no indirect appropriability component of the new unit price.*, thus we can say that indirect appropriation across consumers is not effective anymore.

Indirect appropriation through products

While indirect appropriation through consumers mechanism seems to have been blocked by digitalization, it seems that it is still possible through complementary goods for a given consumer. This branch of literature is quite recent and it stems from the diffusion of peer-to-peer networks such as Napster and their interaction

with profits from concerts and artists' merchandises, the general idea being that when illegal copies are consumed together with a complementary good which cannot be copied, then indirect appropriation is possible.

Two of the main contributions in this field are Krueger (2005) and Connelly and Krueger (2005). Krueger (2005) in detail studies the music industry for Rock and Roll, also defined later as Rock'o'nomics. In this type of economics, there are few unique elements such as the contractual arrangements between bands and promoters. While such contracts normally are long-time and specify a certain quote of royalties and royalty rate for the band which normally failed to even cover the costs of album or single releases[28], what truly determine the bands and authors income are tickets and merchandise. From the Pollstar database and the Encyclopedia of Rock and Roll. Krueger then created a dataset comprising the year each band was formed, the gender of performers, the genre of music and prices on concert tickets and album from 1981 to 2003 and made an analysis on the complementarity between the prices of concerts and the prices of albums and he showed how it is observable a declining in ticket sales and rising price consistent with the market stronger monopoly but inconsistent with a downward shift in demand.

In detail, as an explanation for the inconsistencies between the data and the results of his analysis, Krueger introduced various theories, one of which is the *"Bowie Theory"*. The main concept of this theory can be simplified as follows: as record sales decreased from 1999 to 2002 due to p2p platforms such as Napster, authors were forced to raise the prices of concerts to continue to earn profits, thus record sales and concert tickets are complementary products. The name of this theory derives from the rock and roll singer David Bowie, as he *predicted that "music itself is going to become like running water or electricity" and he advised performers, "you'd better be prepared for doing a lot of touring because that's really the only unique situation that's going to be left"*[29]. From a formal exposition, the copyright

[28] For more detail on those data, see Caves (2000)

[29] quote from Krueger (2005)

owner is a monopolist firm with two complementary outputs. In his work, Krueger denoted concert sales and record albums as good 1 and good 2. The demand for good 1 is defined as $D_1(p_1, p_2)$, while the demand for good 2 is defined as $D_2(p_1, p_2)$. Krueger also considers an independent cost for production of each good defined as $C_1(D_1)$ for production of good 1 and $C_2(D_2)$ for the production of good 2. The firm maximization problem, given this setting, can be rewritten as:

$$\max_{p_1,p_2} p_1 D_1(p_1, p_2) + p_2 D_2(p_1, p_2) - C_1(D_1) - C_2(D_2)$$

Thus, a profit maximizing firm will set a markup of concert tickets under the as follows:

$$\frac{p_1 - C_1}{p_1} = \frac{1}{\epsilon_{11}} - \frac{(p_2 - C_2)D2\epsilon_{12}}{p_1 D_1 \epsilon_{11}}$$

Where ϵ_{11} is the own-price elasticity and ϵ_{12} is the cross-price elasticity of demand. This equation leads to the following contraction: as long as greater attendance to concert leads to an increase in album sales, then firms (the bands in this case) set a price for concert below the single-market monopoly price, but as the technological change known as file sharing erodes the profits from album sales then the price of concert increase is a natural consequence, thus mitigating the profit loss due to digital piracy. Note that similar theoretical and empirical results on the music market have been shown in the theoretical works of Curien and Moreau (2009(1) and 2009(2)), which focus on how the live audience of an artist is connected to the consumption of recorded music through purchases or digital piracy and in the empirical contribution of Bacache, Bourreau and Moreau (2012). In their empirical work, the authors show a positive correlation between the intensity of the live performance of the authors and the favorability they show toward piracy, this holds true for both authors with contract with discographic firms and without contract. An exception are self-released artists who in fact tend to be against piracy as they do not live-perform. They also show that neither age or gender have a significant

effect on tolerance toward digital piracy, while wealth appears to be relevant with wealthier artists being less tolerant to digital piracy.

Another recent contribution was made in 2012 by Mortimer, Nosko and Sorensen. The authors investigate complementarity between concert tickets and album sales once again in an empirical research, finding consistent evidence of indirect appropriation through revenues derived from non-digital complementary products. While the empirical evidence shows that it is true that there is a sales displacement effect on the market for recorded music due to peer-to-peer file sharing, there is also much evidences of positive effects on the market. There is evidence that peer-to-peer technologies lead to broader distribution of music and lead to an increase in the demand for concerts from less-known artists and an increase in album sales for such artists. At the same time, the data showed that there is no effect of peer-to-peer distribution with regards to the earnings derived from concerts for well known artists.

In conclusion, there is solid empirical evidence that whereas there is complementarity between a digital product and a physical product, indirect appropriation is possible due to copyright owners charging a higher price for the physical complementary good.

1.6 Network Effects

Network effects are a type of consumption externalities, based on the fact that *the utility that a consumer derives from consumption depends directly or indirectly on the consumption decision of other consumers*[30]. A classical example of network effect is the diffusion of text editor software: a text editor allows users to produce files of a certain file format. As in spreading information between users, in the text format it is necessary for both consumers to use the same format, it becomes

[30]Peitz and Waelbroek (2006)

intuitive that the more widespread a certain text editor is, the easier to exchange information becomes. From an empirical point of view the previous example is shown in Givon, Mahajan and Muller (1995). In their work, the authors presented a study on software piracy in the UK and showed how digital piracy generated demands for legitimate software. Similar results have been found in empirical studies from Prasad and Mahajan (2003), where the authors confirm how digital piracy accelerates software diffusion amongst consumers.

Another example of consumption externality is the creation of fandom for books or movies. Another form of network effect is the one shown in books and movies, where the creation of a "fandom" community which shares opinion on certain topics leads to a "virality" effect on certain series. The foundation of the connection between digital piracy and network effect was introduced by Conner and Rumelt (1991): in their work, they note how as far as digital piracy is involved, the positive effect of network effect apply to both originals and illegal copies and thus in certain markets the presence of a certain degree of digital piracy may be optimal both for firms and for welfare.

Another relevant contribution is by Takeyama (1994), who introduces the cost of enforcement of copyright protection. In her model, the author uses a Mussa and Rosen approach. She uses a discrete approach, with consumers distributed into two types: high valuation consumers and low-valuation consumers based on consumer sensibility to network effect. Also, instead of a taste parameter θ, the main driver of heterogeneity between consumers is the number of users that consumes the good either legally or illegally N in Takeyama's model. High-valuation consumers will derive utility $N-p$ when consuming the original and utility $\alpha N-c$ when consuming the copy. Instead low-valuation users will extract utility $\epsilon N - p$ when consuming the original good and $\epsilon\alpha N - c$ when consuming the copy, $\epsilon \in [0, 1]$ being the reduced intensity of the sensibility to network effect perceived by consumers. Given these settings, high-valuation consumers will suffer more for the disutility derived

from using a copy [31] than low valuation consumers. These results mean that low-valuation consumers will consider copy and original as substitutes much more than do high-valuation consumers. Also, from the utility extracted by consumers, it is also clear that network effect is perceived with more strength for the original than for the copy. Takeyama shows that the optimal pricing strategy would be to set a price sufficiently low to allow for all consumers to consume the product legally, thus leading to no digital piracy in the market and full exploitation of the network externality. When a firm faces digital piracy, the optimal strategy is to charge a higher price to high-valuation consumers. In these settings, producers will be able to extract the surplus of the high valuation consumers comprising the network effect. Thus, under certain conditions, profits are higher when there is no copyright protection with respect to the profits with full copyright protection. Takeyama also shows that copying can lead to a Pareto improvement in welfare, in comparison to the setting where copying is not possible.

The conclusions from Takeyama (1994) were extended to the duopoly setting by Shy and Thisse (1999). Their results imply that when duopoly is present in a market with network effects, allowing a certain level of digital piracy improves the software network externalities increasing the profits for the duopolists. They also show a positive correlation between increasing network effect and software demand, leading to higher prices and thus to higher profits for the duopolist. The results of Shy and Thisse (1999) were confirmed in the empirical results obtained in King and Lampe (2003) and in De Castro, Balkin and Shephard (2007).

Belleflamme (2003) extended the Russa and Mosen framework for a continuum of users to include network effects shown in the previous Section. In this extension, consumers' valuation for quality θ is increased by the number of consumers which consume the digital goods either legally or illegally N, based on a certain intensity of the network effect defined as $\mu \in [0,1]$. The utility extracted by con-

[31] It is easy to see as $\alpha N > \alpha\epsilon N \forall \alpha, \epsilon$ and N as both $\alpha \in [0,1]$ and $\epsilon \in [0,1]$)

sumers by consuming the digital goods legally thus change to $(\theta + \mu N)q - p$ and the utility extracted by consumers by consuming the digital goods illegally change to $(\theta + \mu N)\alpha q - c$. Belleflamme (2003), given this setting, shows that as long as there are weak network externalities monopolist profits will always be reduced.
Another approach to network effects is shown in Gayer and Shy (2003). In their work, the authors introduce two types of network effect, one based on the number of originals and one based on the number of copies where originals offer higher fixed utility and stronger network effect with respect to the copy. In this setting, they analyze a monopoly model with horizontal differentiation and their analysis yields that where network effects are sufficiently strong for the originals, then there will be an increase in monopolist profits where there are copies in the market.
In general, when network effects are involved in the market, firms may accept end-user copying in the market under the condition that the firm is able to price discriminate between the users of originals and the users of copies and under the assumption that the firm is not able to use a superior discrimination strategy. From a policy point of view, this implies that in a market with network effect it is preferable to allow for weak IP protection, as with weak IP protection both firms and consumers will be better off rather than with strong IP protection. This result conflicts with the one shown in the basic analysis of end-user copying: while in the basic analysis models it is shown that digital piracy lead to a lack of incentive to create or to offer better quality in the long run, when network externalities are involved digital piracy may even act as an incentive to innovate.

1.7 Consumer Information and sampling

Most Information Goods, such as music, books and movies, are experience goods and consumers need experimentation to assess the quality of the Information Good and/or if the product matches their taste. In the time before digitization, sampling

was a common way to transmit information from firms to consumers, but it comes with the costs to transmit such information. Examples of such sampling are movie trailers, listening stations in music stores or simply browsing of a certain book in a book store or in a library. Digitalization made sampling easier, as free sample versions can be distributed quite easily as one of the main characteristics of digital markets is to allow for information transmission at negligible cost[32]. On the other hand, digitalization brought the problem of digital piracy in the market.

Although the potential for illegal copies to have an exposure effect on the market was already perceived and considered by Liebowitz in 1985, digitalization greatly amplified the impact and the range of illegal copying. Ahn and Yoon (2009) extended the Mussa and Rosen approach seen in Yoon (2002), Belleflamme (2003) and Bae and Choi (2006) to allow for sampling effect. In this work, the authors compare a base model for analogical illegal copying with a model inclusive of digitalization and of the sampling effect. The comparison is made through comparative statics on the monopolist profits, consumer surplus and social welfare. While the base model for analogical illegal copying is the same described in Section 1.4.1 for a Mussa and Rosen vertical differentiation with digital piracy, the sampling model introduces sampling as a multiplier of the consumer taste parameter θ defined as $(1+s)$ where $s > 0$ represents the sampling effect of an original and as $(1+t)$ where $t > 0$ represents the sampling effect of a copy. The authors also define α' as the quality degradation for a digital copy, where $\alpha' > \alpha$ as they argue digitalization reduces the quality degradation of copies. The authors also introduce c' as the reproduction cost of copies. Again, the authors argue that $c' < c$ due to digitalization reducing the cost of creating a new copy of the original. Given this setting, the utilities which a consumer derives from consuming an original when there is digitalization and the sampling effect is present is defined as $(1+s)\theta - p$ and the utility a consumer derives from consuming a copy when sampling effect is

[32]see Goldfarb (2019) for more detail on digital economy and its characteristics.

present is defined as $(1+t)\alpha'\theta - c'$. The analysis of comparative statics for the firm profits leads to the conclusion that sampling effect is present and is positive and is able to counteract fully or at least in part the negative sales displacement effect of digital piracy. The analysis on consumer surplus instead yields the same results as shown in the basic models on digital piracy: the effect of digital piracy is always positive on consumer surplus. Last but not least, the comparison between social welfare in the analogic model with illegal copying and the digital model with consumer sampling show that although the effect is ambiguous, it may indeed be positive under certain conditions.

In more recent years the results of Takeyama (1994) on the positive effect of digital piracy on innovation have been confirmed by Choi and Perez (2007). In their papers Choi and Perez study data from Napster and BitTorrent to show how digital piracy led to innovation. To do so, they follow the introduction of a new technology in the market: at the initial stage consumers do not know the new technology and use digital piracy for consumer sampling. At the second stage, adopters interact through communities, giving the developers the ability to extract feedback and insights on the new technology from the adopters. At the third stage, firms develop new products or old products based on the insights gathered in the second stage. In conclusion, the findings from Choi and Perez (2007) not only offer new insights on the relationship between digital piracy and innovation, but also show further effects of the consequence of sampling through illegal copies of the original and not through samples.

Another important contribution in this field is from Gopal, Bhattacharjee and Sanders (2004). In their work, the presence of a peer-to-peer technology has a sampling effect, allowing consumers with a moderate valuation for the product to find what they like of the product itself. This leads consumers that would normally not download the music to do so legally, if the market allows for it, thus increasing the firm profits. The writers also show that this sampling effect is stronger for work

of less known authors.

Peitz and Waelbroeck (2004 and 2006(2)) studied a horizontal differentiation in a multi-product scenario in which consumers are oblivious of the characteristics of a product and are thus forced to choose randomly between products. Peitz and Waelbroeck (2004 and 2006(2)) assume that peer-to-peer technology allows consumers to receive a signal from the goods. Given this signal, consumers are able to self-differentiate with respect to their tastes. The sampling effects in this case allow the firms to select the consumers with higher willingness to pay for the original, and thus firms benefit from the availability of digital copies.

Last but not least, we will take into account the studies by Takeyama in 2004 and in 2009. In her work, the author studies adverse selection models with asymmetric information and considers a durable good and set an intertemporal price-discrimination model, using a the Mussa and Rosen approach. In her setting, the monopolist produces a good which can be of two exogenous qualities (high and low). While the monopolist knows the quality of the good, consumers do not. As the durable good is sold in two periods to a continuum of consumers of unitary dimension, and as all consumers are present in both periods, the asymmetric information leads to an adverse selection problem for high-quality firms when copies are not available in the market.

When copies are available in the market, all consumers will receive less utility from copies than from the original, although the utility they receive is not equal for all consumers. Takeyama introduces two types of consumers: consumers that will receive a higher value from copies and on the other hand consumers that will receive low value from copies. Formally, the utility consumers derive from copying will be defined as $\alpha_i \theta q - c$ for $i = L, H$ and $\alpha_H > \alpha_L$, where α_H is the degradation of quality for consumers with high valuation for copies and α_L is the degradation of quality for consumers with low valuation for copies, where c is the price to obtain the copy. Note that Takeyama sets consumers with low valuation for copying as

captive consumers, as they will never be able to copy. To reach this result, she sets the cost c such that it will be higher than the benefits the consumer will receive for every price and true quality of the good. The author considers both a pooling equilibrium and a separating equilibrium.

The pooling equilibrium leads to intertemporal price discrimination. In the first period the firm will sell its products to consumers with low valuation for copies, while the firm will allow consumers with high valuation for copies to copy. As the copy has lower valuation of the original, in the second period the firm with high-end products will be able to sell its products to consumers with high valuation for copies at a price $(1 - \alpha_H)\theta q$ equal to the difference between the value of the original (θq) and the value of the copy for consumers with high valuation for copies $(\alpha_H \theta q)$. In this case, information transferred to consumers from the sampling effect in the first period allows high quality firms to solve the adverse selection problem and earn profits.

In the separating equilibrium, the firm allows the high quality of the original to be revealed to consumers before the purchase, thus leading to low quality originals to not be available in the market. This result is examined in more detail in Takeyama (2009), where the author studies how when there is asymmetric information, high quality firms would find it more profitable to tolerate digital piracy than to enforce full IP protection.

In conclusion, as consumers face uncertainty due to the characteristic of digital goods being experience goods, digital piracy in a market with asymmetric information may allow consumers to benefit from a sampling effect, thus increasing consumer's surplus and firms' profits, increasing social welfare in both the short and long term, although it is worth noting that this effect is stronger for high quality firms and less known brand/artists.

1.8 New business responses to digital piracy

In the previous Section we saw how digital piracy may allow for consumer uncertainty in a situation of asymmetric information to be solved, acting as an information-pull system. The rise of new distribution channels including the ability to stream and the mass storage capabilities of the internet leads to a change in the way consumers consume, share and transfer Information Goods. Thanks to the new distribution technologies, new firms arise in the market. These firms act as intermediaries, allowing consumers, copyright owners and other stakeholders to connect, in a way which is similar to the role of banks in the financial system. Although these firms use similar technologies, they operate in different ways, giving rise to completely new business models. This thriving and new environment poses a new challenge to economists, and a new flourishing literature has in recent years investigated this area. Belleflamme (2016) offers a detailed and more in depth analysis of the literature on this new world of digital evolution.

As far as the focus of this analysis goes, we will briefly touch upon each of these challenges, show the main research direction in the field and draw the parallel, wherever possible, with those new research topics and digital piracy.

The first research direction is to understand how streaming interacts with normal digital sales. The question is still open, as this is a new field of research. Some articles, such as Aguiar and Waldfogel (2015), Wlömert and Papies (2016) and Hiller (2016) suggest that streaming is a substitute for purchasing. Other studies, such as Aguiar (2015) and Kretshmer and Peukert (2014) suggest that consumer may use free streaming platforms to search for music that conform to consumer taste, then proceed to purchase digital music they discovered in the first phase if it matches their taste, thus leading to an increase in song sales. As far as the relationship between streaming and digital piracy goes, there is little existing research. Two survey-based analyses from Borja, Dieringer and Daw (2015) and Borja and

Dieringer (2016) find a positive correlation between a frequent use of streaming and illegal downloading, Riekkinen (2018) makes an analysis on an online survey, which suggests that subscription video on demand satisfaction is determined by content quality and thus although such services have a small negative effect on attitude toward piracy, they still are not seen as true alternatives sources of video content as they do not offer a complete variety.

The second challenge is to understand how platforms interact with consumers and between themselves. Literature on the two sided market is mainly based on the work of Rochet and Tirole (2003), Armstrong (2006) and Weyl (2010). On the relationship between a two sided market and digital piracy, the work from Rasch and Wenzel (2013) studies the impact of software piracy in a two sided market setting where on one side of the market are developers and on the other side of the market are consumers. The results suggest that in this case although developers benefit from stronger copyright protection, the platform profits will instead decrease.

The third challenge is to understand how these digital platforms manage their business responses. On this topic, Aversa, Hervas-Drane and Evenou (2019) offer a qualitative analysis of the theoretical and practical implications of business model responses to digital piracy and the implication in terms of innovation, diversification and network competition. The two business models which are now widespread are premium subscription and freemium subscription. Sato (2017) constructed a model of menù pricing in a two-sided market to better explain the interaction between the two pricing strategies and its equilibrium characteristics. Instead with regard to how and why a firm chooses premium or freemium, there are important studies by Thomes (2013) and Carroni and Paolini (2016). The relationship between the introduction of the freemium platform and digital piracy is deep. As we argued in Section 1.7, digital goods are often experience goods and one of the reasons consumers use digital piracy is to gain information on a good. Evidence indicates that freemium models act in the same direction, leading to a reduction of digital

piracy in the market. The first work which suggested that free functionality in a software leads to less digital piracy is Cheng and Teegen (1997). In more recent years, the contribution from Halmenschlager and Waelbroeck (2014) showed how online music streaming services act as a deterrent to digital piracy. Nan, Wu, Li and Tan (2016) also analyze optimal pricing for Information Goods under the freemium strategy when in the presence of piracy and network effects on the market. Nan et al. propose a two stage model, using a setting of vertical differentiation à la Mussa and Rosen. In this setting, consumers at the first stage choose between buying a premium version, using pirated goods, using the free version or not using the good at all. Consumers who use free version or pirated goods at the first stage, at the second stage will update their quality perception to reselect at the second stage, thus leading some of those consumers to move from freemium or piracy to premium subscriptions. The presence of a free version is positive in the presence of piracy, as it is more profitable, while digital piracy has a negative impact on the firm even when network effects are present. In conclusion, although some of those approaches seem to be able to reduce Digital Piracy in the market, they do not completely eradicate the problem [33].

1.9 Concluding remarks

Digital piracy is a worldwide phenomenon, spanning across all countries and all digital markets. We can formally distinguish between commercial piracy, which is a counterfeiting criminal activity where an organization sells copyrighted products for profits and end-user piracy, where an individual engages in digital piracy to directly consume a good. While end-user piracy is morally condamnable, there are many reasons why a normally law-abiding citizen violates the law. The literature on the ethics of digital piracy reveals numerous reasons, ranging from a lack of moral

[33] see Tovar et al (2019)

qualities of the individual, to a lack of social pressure against the phenomenon to the necessity to access content otherwise not available in your country.

From an economic point of view, literature on digital piracy started in the early 1980s and has since evolved in various fields and directions. The first direction is to understand the effects of digital piracy on consumers, firms and welfare. This branch of literature uses mainly an approach of vertical differentiation models à la Mussa and Rosen and it offers several insights on the effect of digital piracy on the market. In the short run digital piracy reduces the monopolist's' profits and increases consumer surplus, as it reduces the monopolist's' market power, even leading to a potential positive welfare effect. In the long run, the erosion of digital piracy on profits leads to a lack of incentives to produce new goods or to a decrease in the quality of the goods itself. Whilst this seems theoretically sound and grounded, some empirical evidence seems to underline that this is not always the case. Especially for the music industry, the data seems to suggest that there is no decrease in quality nor in quantity production of music[34].

In literature there are three theoretical arguments that have evolved over the years to explain possible cases in which digital piracy has a positive effect on the firm's profits. Those three mechanisms are known as indirect appropriation, network effects and consumer sampling. We analyzed in detail the characteristics and main contribution in each of those fields.

Indirect appropriation effects may arise through consumers aggregation or through complementary goods. The first mechanism consists of allowing consumers to aggregate in clubs or teams and price-discriminate to indirectly appropriate back for the illegal copies. Although this mechanism is probably sound, digitalization flooded the market with illegal copies, causing this mechanism to stop. The second mechanism allows the copyright owner to charge a higher price for a physical good which is complementary to the digital good. An example in literature is the complemen-

[34]although the quality may be questionable based on personal musical taste.

tarity between digital music and live concert tickets.

Moving forward, the presence of network effects arises when consumers derive higher utility based on other consumers' consumption choice. This mechanism is quite strong in the software industry, but also leads to the creation of "fandom" in other industries such as the music industry or the book industry. In general, literature on Network Effect reaches a consensus that when this effect is present, the negative effect of digital piracy is normally reduced and may even lead to higher profits.

As most digital goods are also experience goods, informational asymmetries may arise when consumers must make a choice. In those cases, digital piracy may lead to a sampling effect allowing consumers to overcome the informational asymmetries and leading to higher profits for the firm. This mechanism is especially strong for high quality firms and for less-known artists.

Last but not least, digitalization is always evolving in time, thus leading to new distribution channels such as streaming, to the rise of intermediaries in the digital market which operate as platforms and to new business models that allow those platforms to operate with profits. While it is clear that legal streaming services have a strong restraining effect on digital piracy, they do not offer enough variety to really lead to a consistent reduction in piracy rates. At the moment, the strongest observed effect of those new technologies on digital piracy is due to the introduction of freemium business model. From the point of view of consumer information, free versions of a product lead to the same result of digital piracy in a model without free samples, also frequently leading consumers to move from freemium version to premium version after trying the quality of the digital product.

To conclude this Chapter, we want to highlight that although end-user digital piracy is a morally reprehensible behaviour, it is an existing and thriving phenomenon in our world, thus we need to find an answer to many unanswered questions on this topic and to better understand how firms may profit from digital piracy on the market. On this topic, in Chapter 2 we will introduce a new theoretical frame-

work for indirect appropriation across complementary Information Goods which includes digital piracy as an endogenous variable and in Chapter 3 we will extend said framework to account for network externalities.

Chapter 2

A model on Indirect appropriation across Complementary Information Goods

Abstract

Digital piracy is a worldwide, highly pervasive phenomenon across countries and sectors, with social costs estimated in billions USD[1]. This phenomenon mainly takes the form of End-User digital piracy. In End-User piracy a single consumer illegally consumes a good without paying the due price to the Copyright Owner. The industries that suffer the most from End-User piracy are those based on Information Goods, such as the motion pictures industry, the music industry, book publishing and the software industry. To counteract the effects of Digital Piracy firms have come up with various strategies, one of which is the use of complementary products to allow for indirect appropriation. Numerous companies base their business model on the creation of franchises which complete and enhance the consumption expe-

[1]Frontier Economics' 2017 Economic Impacts of Counterfeiting and Piracy Report for BASCAP and INTA and 2016 UK intellectual property office report

rience offered by the Information Good. In this work we present a general model for vertical differentiation and digital piracy à la Mussa and Rosen (1978) where a monopolist offers an Information Good in a digital market affected by end-user digital piracy together with a complementary good, thus analyzing and uncovering some of the underlying mechanics of this type of markets.

2.1 Introduction

As far as economic literature is involved, intellectual property rights (IPR) are an important legilative instrument to protect and incentivize innovation. IPR allow the balance between two problems: under-production and under-utilization of Intellectual Products.

Under-utilization is implied by the characteristic of non-excludability of intellectual products, and it is solved in literature by allowing a legal right - the IPR - to exclude competitors from the market, thus creating a legal monopoly. At the same time, such monopoly grants firms with market power, which converts into higher prices and thus lead to under-utilization of the good. To balance those two problems, IPR legal monopolies are not eternal, but instead have a set duration and scope.[2].

From a Copyright Owner's point of view, Copyright grants access to a legal monopoly over the Information Good and all connected products and is it a necessary and strong instrument for earning profits and continuing to create newer innovations.

Academic and non-academic literature has richly documented the extensive and intensive lobbying efforts of firms on legislators to allow for stronger, longer and stricter IPR laws[3]. As shown in detail by Schlackman (2018), the effects of lobbying are traceable and clear in the US copyright length history: the 1790 Copyright Act enforced a 14-years renewable term for Copyright for a maximum of 28 years,

[2]Belleflamme (2012) delve deeper in those literature concepts.

[3]The process of lobbying is described in detail in: Ota (1998), Greenhouse (2002) and Schlackman (2018).

today Copyright length is set as the life of the author plus 70 years for works authored by single authors, while the Copyright length for corporate works is set at 95 years from the first publication or 120 years from the creation of the Information Good, whichever is shorter.

One of the main drivers for requesting longer and stricter Copyright protection in modern time is the phenomenon known as *digital piracy*. In literature there are various ways to define the phenomenon of Digital Piracy. We choose the definition offered in Belleflamme and Peitz (2014) as follow: Digital Piracy is the act of reproducing, using or distributing information products in digital formats and/or using digital technologies, without the authorization of their legal owners, where information products are a type of intellectual property (IP) products such as books, music, movies or paintings.

Faced with this problem, firms united in organizations such as the Alliance for Creativity and Entertainment, the Intellectual property Owners Association and the Recording Industry Association of America while law enforcement agencies all over the world are sparing no effort trying to reduce digital piracy through DNS blocking and online illegal distribution platforms crackdown.

Notwithstanding those efforts, digital piracy is still highly pervasive across countries and sectors, and it is a multi-billion-dollar industry that continues to grow and it is clear from the data[4] that digital piracy creates a sales displacement in the IG industries as it offers a convenient and cost-effective method to access desired content to consumers. Firms operating in the IG industries have to strive and thrive through the harshness of this environment, bearing the brunt of this phenomenon. To allow survival, the markets have seen the birth of various strategies. Although in recent years the creation of new business models such as the one introduced in Chapter 1 Section 8 seems to have a restraining effect on the level of digital piracy in the market, those approaches still do not lead to complete eradication of digital

[4]Frontier Economics' 2017 Economic Impacts of Counterfeiting and Piracy Report for BASCAP and INTA and 2016 UK intellectual property office report.

piracy from the market [5].

In this work, we will focus on one of the main strategies, applied across all the industries that base their earnings on brand image and franchises, such as the movie industry, the physical game industry, the videogames industry, the music industry and the printed book industry, but also the sport industry: the creation of complementary physical or Information Goods to enhance the consumption of a certain Information Good and thus allow for indirect appropriation across products.

Since the early 1980s, copyright owner firms have opted for differentiation of activities through creation of franchises. Since most IGs are experience goods, it is possible to create Alternative Goods that complement the consumer consumption experience, and as Alternative Goods are also covered by the copyright protection umbrella, they are under the monopoly of the copyright owner. Such type of Alternative Goods may be both physical goods or other IGs connected to the original IG.

The best exemplification of this strategy is the Disney Company. The Disney Company is not only one of the biggest copyright owners in the whole history of mankind, but it also offers the *Disney Experience* to its consumers. While a good portion of Disney profits are connected to box office outcomes, their activities include themed parks, theatrical representations, DVDs, gadgets and services to consumers. The 2018 Disney Company Annual Report to Investors classifies activities of the company in four main segments: *media networks*, *park and resorts*, *Studio Entertainment* and *Consumer Products and Interactive Media*. The media network segment generated in 2018 revenues for 24.500 millions USD, of which 6625 millions USD are the segment operating income with a profit margin ratio of around 27%. The park and resorts segment generated revenues for 20296 million USD, of which 4469 millions USD are the segment operating income, leading to a profit margin ratio close to 22%. The Studio Entertainment segment shows revenues for 9987 million

[5] More detail on this topic may be found in Tovar et al (2019)

USD, and segment operating income for 2980 million USD, which translates in a profit margin ratio nearing 30%. The last segments, Consumer Products and Interactive Media generated 4651 million USD and a segment operating income of 1632 millions USD, with a profit margin ratio close to 36%. When analyzing this data, it is easy to spot how of the total income of around 59434 millions USD about 59% (34934 millions USD) derives from alternative markets. Also, note that due to the higher profit margin ratio present in most alternative activities, of the total profits (15706 millions USD) more than 60% (9081 millions USD) derives from alternative markets.

Notice that this phenomenon is still in development as of today. As an example, in August 2019, during the biennial convention called D23, the Disney Company announced how two of its newest franchises will join the Disney's offer for entertainment. At the same time, the creation of the new Avengers Campus at Disney California Adventure and in Disneyland Paris was announced, promising consumers "to live the Marvel Avenger franchise experience". Walt Disney World Resort *Star Wars: Galactic Star Cruiser* was also launched. It invites guests to embark on a two night adventure aboard a glamorous starship[6] in the Star Wars Franchise.

Another example of the importance of the Alternative Goods is the *Wizarding World* franchise by J.K. Rowling: from the original Harry Potter book saga, it generated the third-highest-grossing film franchise of all time (behind the Marvel Cinematic Universe and Star Wars), numerous video games, dedicated Lego sets, action figures and gadgets such as wands, capes, notebook and even a theme park from Universal Studios.

Other examples are famous sagas and/or franchises, such as the Song of Ice and Fire saga from J.R.R. Martin which encompasses not only the famous HBO tv series, but also a plethora of gadgets, action figures and branded clothes. From the software industry, some relevant examples are the Assassin's Creed saga, the Halo

[6]surce: D23 website at - http://www.d23.com

Saga, the Mass-Effect saga and the Dragon Age saga which offer numerous Alternative Goods such as Action Figures, branded clothing, stuffed toys and so on. In the music industry we can consider live performances, t-shirts and collector's editions and digital distribution as Alternative Goods and there are even more famous examples, such as the Final Fantasy saga or Nintendo's Amiibos.

This strategy does applies to movies, books, softwares and music. Since the early 1980s bands and songswriter not only sold CDs and albums, but also merchandise such as t-shirts and live performances tickets. For the sport industry, the memorabilia from sport teams such as t-shirts, balls or even toys allow for indirect appropriation from illegal streaming of games.

As digital piracy exists in this world and as big firms and authors frequently use Alternative Goods in their activities, we created this model with the intent of gaining insights on the role of digital piracy in the market. We aimed to discover how and in what conditions digital piracy can become a strategic instrument in the hands of firms to maximize their profits, to regulate or to better understand the mechanism underlying indirect appropriation across products.

To reach our aim, we create a variation to the Mussa and Rosen model for vertical differentiation, similar to the one introduced in Section 1.4.1. In our model a copyright owner offers a IG in the intellectual market, and also offers a complementary good in an alternative market.

As the Copyright extends to both the IG and the Alternative Good, the copyright owner is a monopolist in both markets. We define the primary market as the market where the IG is sold. We define the alternative market as the market where the Alternative Good is sold.

For simplicity's sake, we will consider that the alternative market trades only in royalties and thus there is no production cost for the Alternative Goods. This assumption does not affect the generality of the model and it is coherent with what happens in the real world. A clear cut example can be seen in the Lego Company.

The Lego company acquires IP from franchises paying royalties to multiple copyright owners to be allowed to produce Lego sets of said franchises. Another example in the digital distribution industry are platforms such as Netflix or Amazon Prime: if they want to offer a certain movie in their catalogue, then they will have to pay a certain royalty to the copyright owners of the movie.

At the other end of the spectrum, we have consumers. Based on their taste for the quality of the Information Good, consumers are uniformly distributed in a segment of unitary dimension. Notice that as we assume that both the original and the copy are of the same quality, for simplicity sake we set the quality of the good $q = 1$. Based on their taste for the quality of the Alternative Good, consumers are discretely distributed. Fraction λ_b of consumers is interested in consuming the bundle formed by the digital good and the Alternative Good, while fraction λ_0 is interested only in consuming the digital good. This is a relevant assumption as it implies complementarity between the digital good and the Alternative Good and the fact that a consumer will be interested in buying an Alternative Good as a complement of the experience granted by the digital goods. This concept spawns from the idea that a consumer will be willing to pay for an action figure of a character only if the consumer knows the character itself. Another example is how consumers will not choose to spend their vacation in a themed Avengers resort if they are not fans of the Marvel Universe.

Our results show that in a market where piracy exists, then it also exists an equilibrium level of piracy such that the copyright owner can maximize profits. Also, we will show that the structure of the model allows for a "virality effect" and how this effect is amplified by digital piracy, thus helping to explain the success of tv series such as Games of Thrones[7]. Also, the results offer insights into the classical digital piracy theory on the very definition of legal monopoly and its usefulness.

[7]The following interviews on the topic may be interesting on this point - https://www.npr.org/2013/04/07/176338400/pirates-steal-game-of-thrones-why-hbo-doesnt-mind and https://www.washingtonpost.com/news/the-switch/wp/2013/08/09/game-of-thrones-exec-says-piracy-is-better-than-an-emmy-he-has-a-point/

Last but not least, we will make an argument for a possible way to allow firms for a personalizable fine-tuning of the digital market, transforming digital piracy in a real instrument to maximize profits in the hands of copyright owners thus leading to a more flexible law enforcement system that may allow firms to choose the strategy that allow for profit maximization.

2.2 Related Literature

There are various branches of literature related to this work: literature on basic analysis of digital piracy, literature on indirect appropriation, literature on complementary goods, literature on IP protection enforcement costs, literature on commercial piracy and literature on mix and match models.

From the literature on basic analysis of digital piracy, we use a specification of the Mussa and Rosen (1978) framework. An in depth analysis of this framework and of its application in the digital piracy literature is presented in Section 1.4.1 of this thesis. On this topic, the contribution from Yoon (2002), Belleflamme (2003) and Bae and Choi (2006) is relevant to our work. We will distance this research from those works for various reasons. In their works, those authors considered the quality of the original being different from the quality of the copy, whereas in our model we will consider both goods to be of the same quality and that such quality is equal to one. This assumption is coherent with the new digitalization technologies which allow for perfect copies of the originals. The second main difference between our work and the previous work is that the variable α is not a degradation of quality of the good alpha, but instead will represent the inverse probability of being caught by law enforcement (which is equal to $(1-\alpha)$). Notice that α still degrades the consumption experience. Although we might like to believe that the end-user pirate is an otherwise law abiding citizen with normal moral standards, and thus will at least feel some guilt while enjoying an illegally downloaded movie or at least as the

discomfort of knowing that they may be caught by law enforcement for their illicit behaviour, we will instead took a lesser moral and more realistic point of view: we can see α as the "bothersome" time expenditure needed to find and download the illegal copy. Because the digitalization revolution allows for copying at negligible costs, in our model there will be no cost of reproduction c. Instead, when illegally downloading the goods, consumers will face the probability of being caught by the police $(1-\alpha)$ and, in this case, being forced to pay a fine M.

As far as literature on indirect appropriation across products is involved, the closest reference is the Bowie theory presented in Krueger (2005). While both theories allow for complementary goods to exist in the market, Krueger does not directly model digital piracy in his Bowie theory. Our model will thus be completely different not only in structure but also in content, as it will be more general and will allow for direct observation of the effect of digital piracy. On the other hand, our work is fully theoretical and does not include any data.

From literature on commercial piracy we will take into consideration the contribution of Yao (2005). In his work, he uses a vertical differentiated model à la Mussa and Rosen where he studies the competition between an incumbent copyright owner and one potential pirate entrant. From this contribution, we took the idea of using $(1-\alpha)$ (ϕ in his notation) as the probability of being caught by law enforcement. In his work, Yao also shows that the probability of being caught acts as an indicator of the strength of IP rights enforcements of the market. Thus, conversely, the negative of the strength of IP rights enforcement in the market α ($(1-\phi)$ in his notation) can be viewed as the level of piracy allowed in the digital market: the more piracy there is in the market, the higher the probability of consuming the good without being caught is.

From literature on IP right enforcement, we took inspiration from Slive and Bernhardt (1998). In their work, the authors present a price discrimination model with network externalities to explain why there is low expenditure on anti piracy mea-

sures in the market. In their model, they consider a linear cost function for digital piracy enforcement, as we do in our model.

Our model shares similarities with mix and match models such as Matutes and Regibeau (1988), Economides (1989) and Economides and Viard (2007). While in those models the typical assumption is that neither good is valuable without the other under a certain proportion, in our model we assume that while the Information Good is always valuable for consumers, the Alternative Good is valuable only when the Information Good is also consumed. This particular mechanic is already presented in Economides (1989) and more recently in Economides and Viard (2007). Also, similarly to Economides and Viard (2007) we will place consumers in a square based on their taste parameter for the Information Good and for their taste parameter for the Alternative Good. Unlinke the model of Economides and Viard (2007), consumers will have a discrete taste for the Alternative Goods, where consumers with valuation for the Alternative Good λ_0 will be interested in buying only the Information Good and consumers with valuation for the Alternative Good λ_b will be interested in buying the bundle of the Information Good and the Alternative Good.

In conclusion, our model is a model on End-User piracy, where we study indirect appropriation across products for a monopolist which offers an Information Good on the digital market and an Alternative Good, which may be a physical or another Information Good, in an alternative market. This model takes in lots of insights from various neighbouring literature, to allow us to create a complete although simple set of mechanisms, to allow for better understanding of this type of markets. Also, this work should offer a general theoretical framework which extends the already existing ones in this field of research to include digital piracy as an endogenous variable.

2.3 The model

In our end-user digital piracy model we consider two markets: one for the Information Good (good I), also known as a primary market, and one for an Alternative Good (good A), also known as an alternative market. In these markets the copyright owner due to the effects of copyright offers both goods as a monopolist, while consumers will generate a demand for both goods based on their utility functions. For the primary market we consider a very simple market for an Information Good supplied by a Copyright owner à la Mussa-Rosen for a continuum of consumers of unitary dimension. The potential users are characterized by their valuation θ for the Information Good[8]. We assume θ to be uniformly distributed on the unitary interval [0,1].

Consumers in the primary market may choose to buy the legitimate product at price p_I or he/she can acquire an illegal product through digital piracy. If a consumer chooses to consume the good illegally (thus becoming a "digital pirate") a probability $(1-\alpha)$ to be caught in the act, where $\alpha \in [0,1]$ is by inverse deduction the probability of not being caught while illegally consuming the product. If caught, the consumer will have to face a fine M set by the government or by a copyright enforcement agency. Note that due to the results shown in Yao (2005), the variable α can be considered as a proxy measure of the level of digital piracy in the market. This result implies that the higher the digital piracy is in the market, the lower the probability to be caught, which is a logical implication.

[8]This is a special case of a Mussa and Rosen settings, where θ is the consumer valuation for quality of the good and is normally associated with the quality q of the base good. As one of the consequences of digitalization is the availability of perfect copies of the original, we can consider that in this case the quality of the copy and of the original are the same and are set as $q = 1$.

A consumer, while consuming the Information Good will derive the following utility:

$$U = \begin{cases} \theta - p_I & \text{if the consumer buys the digital good} \\ \alpha\theta - (1-\alpha)M & \text{if the consumer consumes illegally the digital good} \\ 0 & \text{otherwise} \end{cases}$$

In the alternative market we consider a simple market for an Alternative Good, supplied by a third party producer through a franchise or licensing agreement.[9] Copyright Owners in this market will receive a royalty p_A due to such agreement. Notice that the Alternative Good will generate a positive utility only if consumed as a bundle with the Information Good[10]. In the alternative market, consumers are dichotomous[11] based on their valuation of the Alternative Good ψ. Notice that we consider the consumers' valuation of the Alternative Good ψ as common knowledge for the copyright owner for simplicity's sake. Also, notice that consumers' fraction $\lambda_0 = \lambda$, for $\lambda \in [0, 1]$, will never be interested in the Alternative Good, while fraction $\lambda_b = 1 - \lambda$ of consumers may be interested in buying a bundle of the Information Good and the Alternative Good.

While consuming the Alternative Good, the consumers will derive utility as defined

[9] This assumption simplifies the model while not giving any loss of generality. Krueger (2005), Krueger and Connolly (2005) and Goldfarb (2019) offers insights on how licensing agreements are common in the digital industry we are taking into consideration.

[10] This assumption is the main difference between our models and normal complementary product models. This type of assumption has already been introduced in the works of Economides (1989) and Economides and Viard (2007).

[11] We choose to have a dichotomous approach as our main objective is to create a basic framework to understand the effects of endogenous digital piracy in an indirect appropriation across products. On this account, while on one hand the choice of continuity over dichotomy doesn't appear to yield a positive contribution on the quality of the conclusions we reach in the model (as the focus is not pricing strategies in the alternative market but only the interaction that the existance of the alternative market bring in the mix of digital piracy and Information Goods), the choice of continuity over dichotomy would instead impact heavily on the complexity of calculus.

in the following utility function:

$$V = \begin{cases} \psi - p_A \text{ if the consumer buys the Alternative Good} \\ 0 \qquad \text{otherwise} \end{cases}$$

Notice that in the following sections we will also evaluate a variation of this model, which we call a benchmark model. In the benchmark model there is no piracy in the market ($\alpha = 0$). This will allow us to understand the underlying basic mechanics of the model prior to the introduction of piracy and thus to more accurately define what effect piracy has and where.

2.3.1 Consumers' behaviour with End-User digital piracy

Notice that a consumer indexed by their valuation for the Information Good (θ) will buy the legitimate product under the condition that the utility he receives from buying the Information Good is higher than the utility he would receive from copying it. We define $\tilde{\theta}$ as the consumer who is indifferent between consuming the good legally or illegally. As $\theta - p_I = \alpha\theta - (1-\alpha)M$, we can find the cutoff:

$$\tilde{\theta} = \frac{p_I}{1-\alpha} - M \tag{2.1}$$

When a consumer is indexed only by their valuation for the Information Good (θ), he will illegally consume the product under the condition that the utility he receives from illegally consuming the good is higher than the utility he receives from not consuming the good at all. We define $\hat{\theta}$ as the consumer indifferent between consuming the good illegally or not consuming the good at all. As $\alpha\theta - (1-\alpha)M = 0$ then we can find the cutoff:

$$\hat{\theta} = M(\frac{1}{\alpha} - 1) \tag{2.2}$$

From those cutoff points, and based only on their valuation for the Information Good (θ), we can define consumers' type θ_0 if the consumers have $\theta \in [0, \hat{\theta})$, consumers' type θ_p if the consumers have $\theta \in [\hat{\theta}, \tilde{\theta})$, and consumers' type θ_b if the consumers have $\theta \in [\tilde{\theta}, 1]$.

When indexing consumers for their valuation of the Alternative Good (ψ), we face a dichotomous possibility. Consumers type $\lambda_0 = \lambda$ for $\lambda \in [0, 1]$ will never be interested in buying the Alternative Good and consumers type $\lambda_b = 1 - \lambda$ if they are willing to consume a bundle of both the Information Good and of the Alternative Good.

As we consider the double heterogeneity, we will index consumers by both their type $\{\theta, \lambda\}$. Thus in the indirect appropriation model there are five relevant consumer types, based on their combined valuation $\{\theta, \lambda\}$ [12]:

- Consumers type $\{\theta_0, \lambda_0\}$ will not consume either the Information Good or the Alternative Good.

- Consumers type $\{\theta_p, \lambda_0\}$ will prefer to consume the Information Goods illegally in the primary market and are not interested in consuming the Alternative Goods.

- Consumers type $\{\theta_b, \lambda_0\}$ are willing to legally consume the Information Good in the primary market but are not interested in consuming the Alternative Good.

- Consumers type $\{\theta_p, \lambda_b\}$ will prefer to consume the Information Goods illegally in the primary market, but are interested in consuming the Alternative Goods.

[12] We specify relevant as by logic there should be six types of consumers. As a consequence for our assumption on the complementarity of the Alternative Good and the Information Good, it is not possible to consume the Alternative Good if it has not been consumed the Information Good. Under this assumption, consumers type $\{\theta_0, \lambda_b\}$ would have the same behaviour of consumer type $\{\theta_0, \lambda_0\}$ not buying any of the goods, thus they can be treated as a unique uniform group based on their behaviour.

- Consumers type $\{\theta_b, \lambda_b\}$ are willing to legally consume the Information Good in the primary market and are interested in consuming the Alternative Good.

Notice that for piracy to exist in the market, we need to verify the condition $\tilde{\theta} > \hat{\theta}$. We will analyze this condition further in Subsection 4.7.

2.3.2 Consumers' behaviour in the benchmark model

As noted above, in the benchmark model we set that piracy is not allowed in the market, setting $\alpha = 0$. In this case, a consumer can only choose to buy or not to buy the Information Good, thus he will derive the following utility:

$$U = \begin{cases} \theta - p_I : \text{if the consumer consumes the digital good} \\ 0 \qquad : \text{otherwise} \end{cases}$$

Notice that a consumer indexed by his valuation for the Information Good (θ) will buy the product under the condition that the utility he receives from buying the Information Good is higher than the utility he would receive from not consuming it. We define $\tilde{\theta}$ as the consumer indifferent between consuming the good legally or not consuming the good at all. Given the aforementioned utility function, we will have $\theta - p_I = 0$, obtaining the cutoff:

$$\tilde{\theta} = p_I \tag{2.3}$$

From this cutoff point, based only on a consumer valuation for the Information Good (θ), we define consumers type θ_0 when $\theta \in [0, \tilde{\theta})$ and consumers type θ_b when $\theta \in [\tilde{\theta}, 1]$. As in the alternative market there is no piracy in the full model for indirect appropriability, we have no change in the type of consumers based on their valuation for the Alternative Good. So, in both the benchmark model and in the complete model we will have two types of consumers based on their dichotomous

preference toward the Alternative Good: consumers type $\lambda_0 = \lambda$ for $\lambda \in [0,1]$ if they are not interested in consuming the Alternative Good at all, while they are defined as type $\lambda_b = 1 - \lambda$ if they are willing to consume a bundle of both the Information Good and of the Alternative Good. Given this setting, when we index our consumers by both their type $\{\theta, \lambda\}$. Thus, in the benchmark model, there are three types of consumers given their valuation $\{\theta, \lambda\}$:

- Consumers type (θ_0, λ_0) will not consume either good.
- Consumers type (θ_b, λ_0) will be interested in consuming the Information Good but are not interested in consuming the Alternative Good.
- Consumers type (θ_b, λ_b) will be interested in consuming both the intellectual and the Alternative Goods.

Notice that the main difference between the consumers' behaviour in the benchmark model and in the digital piracy model is that $\alpha = 0$, thus consumers will not have the choice to pirate. In other words, we have that in the benchmark model $\tilde{\theta} = \hat{\theta}$. The condition for piracy to be allowed in the model is that $\tilde{\theta} > \hat{\theta}$. We can rewrite this condition as: $M < \frac{p_I \alpha}{1-\alpha}$. As in the real world piracy does exist, we consider this condition to be always respected for our conclusions and elaborations on the indirect appropriation model.

2.3.3 Copyright Owner's Behaviour with End-User Digital Piracy

The Copyright Owner's monopolistic problem in the Indirect Appropriation model is to set the price for the Information Good (p_I), the royalties for the Alternative Good (p_A) and the piracy level in the market (α) such that the copyright owner's profits π can be maximised, given the costs related to producing the Information Good (C_I), the legal cost to sign the royalty agreement (C_A) and the cost to enforce

piracy (C_α). We consider C_I to be a fixed cost because once produced the cost of copying a digital good is equal to zero or otherwise negligible, as shown in Goldfarb and Tucker (2019). We consider C_A as a fixed cost as it represents the search and legal cost related to finding the third party producer for the Alternative Good. We consider C_α as the linear cost related to enforcing piracy in the digital market. We model it as a linear cost coherently with what is shown in Slive and Bernhardt (1998) and for simplicity's sake we set it as $C_\alpha = \alpha$.

The monopolist problem can thus be written as:

$$\max_{p_I, p_A, \alpha} \pi = D_I p_I + D_A p_A - C_I - C_A - C_\alpha \tag{2.4}$$

Where D_I is the demand for good I and D_A is the demand for good A.

2.3.4 Copyright Owner's Behaviour in the benchmark model

In the benchmark model there is no digital piracy in the market as we assumed $\alpha = 0$. Thus, the Copyright Owner maximizes his profits function only based on the prices in both markets and pays no cost to enforce a certain level of piracy in the market. Given this setting, the monopolist problem for the benchmark model can be written as follows:

$$\max_{p_I, p_A} \pi = D_I p_I + D_A p_A - C_I - C_A \tag{2.5}$$

Where D_I is the demand for good I and D_A is the demand for good A.

2.3.5 Timing

The timing of the game is described as follows. In the first period the Copyright Owner will decide the equilibrium level of piracy to allow in the market to maximize its profits. In the second period the Copyright Owner will set the monopolistic

prices for both the Information Good and the Alternative Good. In the third period consumers in both markets will make their consumption choice.
As in the benchmark model there is no choice of digital piracy, we will consider only a first period in which the copyright owner decides the equilibrium monopolistic prices for both the Information Good and the Alternative Good and a second period at which consumers in both markets make their consumption choice.

2.4 Equilibrium Analysis

In this Section we analyze both the benchmark and the end-user digital piracy model. To solve both problems we use backward induction. For the sake of clarity, we derive the results of the benchmark model first and then we derive the results from the end-user piracy model. Following the timing of our model, in the next Subsection 4.1 we will analyze how consumers' behaviour aggregates to form the demand functions for the benchmark model. Then in Subsection 4.2 we will find the equilibrium pricing strategy for the monopolist. In Subsection 4.3 we will then draw out an analysis of the results from the Benchmark Model. In Subsection 4.4 we will then analyze how consumers' behaviour generates demand in both markets, and in Subsection 4.5 we will find the equilibrium pricing strategy for the monopolist. In Subsection 4.6 we will evaluate the equilibrium piracy level for the market. In Subsection 4.7 we will discuss the existence condition for piracy to be allowed in the model. The policy implications of these results will be analyzed in Section 7.

2.4.1 Consumers' consumption choice in the Benchmark Model

Solving per backward induction, in time 2 consumers' will make their consumption choice. Thus, we will take into consideration the consumers' strategies $\{\theta, \lambda\}$ in the benchmark model, to derive the demand function for the Information Good and the

secondary good given the consumers' utility functions as follows:

$$D_{\{\theta,\lambda\}} = \begin{cases} D_{\{b,0\}} \equiv \lambda(1-\tilde{\theta}) & \text{generated by consumers type } \{\theta_b, \lambda_0\}, \text{ for the Information Good.} \\ D_{\{b,b\}} \equiv (1-\lambda)(1-\tilde{\theta}) & \text{generated by consumers type } \{\theta_b, \lambda_b\}, \text{ for both goods.} \end{cases}$$

Thus, the monopolist will face, for good I, demand:

$$D_I = \lambda(1-\tilde{\theta}) + (1-\lambda)(1-\tilde{\theta})$$

Grouping by common factor and knowing that $(1-\lambda)+\lambda = 1$, we can rewrite it as:

$$D_I = (1-\tilde{\theta}) \tag{2.6}$$

While the Copyright Owner will face, for good A, demand:

$$D_A = (1-\lambda)(1-\tilde{\theta}) \tag{2.7}$$

We can summarize this result as follows. In time 2, in the benchmark model, the consumers' consumption choice generate:

- For good I, demand: $D_I = (1-\tilde{\theta})$.
- For good A, demand: $D_A = (1-\lambda)(1-\tilde{\theta})$

2.4.2 Copyright Owners' optimal pricing strategy in the Benchmark Model

Lemma 1 *Solving per backward induction, in time 1, given the consumers' consumption choice stated previously:*

- *the equilibrium price for the Alternative Good s.t. the Copyright Owners profits are maximized is:* $p_A^* = \psi$.

- *the equilibrium price for the Information Good s.t. the Copyright Owners profits are maximized is:* $p_I^* = \frac{1}{2}(1 - \psi(1 - \lambda))$.

Thus, the equilibrium profits given the equilibrium price for the Information Good and the Alternative Good are: $\pi^* = \frac{1}{4}(1 - \psi(1 - \lambda))^2$

From Subsection 3.4 we recall Equation (2.5) which expresses the copyright owner monopolistic problem for the benchmark model:

$$\max_{p_I, p_A} \pi = D_I \ p_I + D_A \ p_A - C_I - C_A$$

From the results shown in Section 3.2 we recall Equation (2.3):

$$\tilde{\theta} = p_I$$

We assume, without loss of generality and for simplicity's sake, that $C_I = 0$ and $C_A = 0$.

As we assumed in Section 3 that the consumers' valuation of the Alternative Good ψ is common knowledge for the Copyright Owner, the copyright owner will be able to extract the full consumer surplus and set the maximum price when $V = 0$. As we know $V = \psi - p_A$ and as the Copyright Owner maximizes his profits when $\psi - p_A = 0$ it becomes trivial to calculate the equilibrium price for the alternative market as:

$$p_A^* = \psi \tag{2.8}$$

Moving forward in our analysis, we can substitute (2.6), (2.7) and (2.8) into (2.5). This allows us to rewrite the copyright owner monopolistic problem as:

$$\max_{p_I} \pi = (1 - p_I)p_I + (1 - \lambda)(1 - p_I)(\psi) \tag{2.9}$$

The first-order condition for profit maximization requires $\frac{\partial \pi}{\partial p_I} = 0$, so we take the

first derivative of the profit function, obtaining:

$$\frac{\partial \pi}{\partial p_I} = 1 - 2p_I - (1-\lambda)\psi \tag{2.10}$$

Setting (2.10) equal to zero and then solving for p_I, we find the equilibrium monopolistic price for the Information Good:

$$p_I^* = \frac{1}{2}(1 - \psi(1-\lambda)) \tag{2.11}$$

Also, given (2.8) and (2.11), we can substitute the equilibrium values into the profit function (π) to find the profit in equilibrium given the equilibrium prices (π^*):

$$\pi^* = \frac{1}{4}(1 - \psi(1-\lambda))^2 \tag{2.12}$$

2.4.3 Benchmark Model discussion

The benchmark model allows us to draw the baseline on what to expect from the indirect appropriation model in the next step, as it introduces the basic mechanics between the two markets and how the fraction of consumers interested in buying only good I (λ) and the consumers' valuation for the Alternative Good (ψ) affect the equilibrium price in the primary market and the copyright owner's equilibrium profits when piracy is not allowed in the market.

To study the effects of a marginal increase in the fraction of consumers that want to consume only the Information Good $\lambda_0 = \lambda$ on the equilibrium price in the primary market, we take the first derivative of (2.11) with respect to λ. We obtain:

$$\frac{\partial p_I^*}{\partial \lambda} = \frac{\psi}{2}$$

As this function is positive for all values of $\psi \in [0,1]$,a marginal increase in the fraction of consumers that want to consume only the Information Good has a *ceteris paribus* positive effect on prices. Obviously, on the other hand, the fraction $\lambda_b = 1 - \lambda$ has a negative effect on prices. This effect is also intuitive as λ has a direct, negative effect on the demand for the Alternative Good[13]. Given the lower demand in the alternative market, the Copyright Owner will have an incentive in increasing prices in the primary market.

To study the effects of the consumers' valuation for the Alternative Good ψ on the equilibrium price for the Information Good, we take the first derivative of Equation (2.11) with respect to ψ. We obtain:

$$\frac{\partial p_I^*}{\partial \psi} = -\frac{1}{2}(1-\lambda)$$

As this function is negative for all values of $\lambda \in [0,1]$, we know that a marginal increase in consumers' valuation for the Alternative Good has a *ceteris paribus* negative effect on the equilibrium price in the primary market.

This result is intuitive as an increase in consumers' valuation for the Alternative Good directly translates into an increase in the Alternative Good price[14]. As a direct consequence, the Copyright Owner will have incentive to increase access in the alternative market and to do so, he will decrease the price in the primary market.

Moving forward, to study the effects of the consumers' valuation for the Alternative Good ψ on the equilibrium profits, we take the first derivative of (2.12) with respect to ψ. We obtain:

$$\frac{\partial \pi^*}{\partial \psi} = \frac{1}{2}(1-\lambda)(1+\psi(1-\lambda))$$

This function is positive for all values of $\lambda \in [0,1]$ and $\psi \in [0,1]$. Thus, a marginal increase in consumers' valuation for the Alternative Good ψ will *ceteris paribus*

[13] as shown in Equation (2.7): $D_A = (1-\lambda)(1-\tilde{\theta})$.
[14] as shown in (2.8), $p_A^* = \psi$.

cause a marginal increase in the equilibrium profits for the copyright owner.

To study how a marginal increase in the fraction of consumers λ affects the equilibrium profits, we take the first derivative of (2.12) with respect to λ. We obtain:

$$\frac{\partial \pi^*}{\partial \lambda} = -\frac{1}{2}\psi(1+\psi(1-\lambda))$$

As this function is negative for all values of $\lambda \in [0,1]$ and $\psi \in [0,1]$, we know that a marginal increase in the fraction of consumers interested only in the Information Good will translate in a decrease in equilibrium profits for the Copyright Owner. Thus, whenever consumers' valuation for the alternative market (ψ) increases, *ceteris paribus* we have a lower equilibrium price in the primary market for the Information Good or higher equilibrium profits.

To study the effect on profits when both variables increase, we take the cross derivative of the equilibrium profits (12). Notice that as both ψ and λ are continuous, we can apply the Schwarz Theorem and thus we have symmetry of the second derivatives, thus $\frac{\partial^2 \pi^*}{\partial \lambda \partial \psi} = \frac{\partial^2 \pi^*}{\partial \psi \partial \lambda}$. We obtain that:

$$\frac{\partial^2 \pi^*}{\partial \psi \partial \lambda} = -\frac{1}{2}\psi(1-\lambda) - \frac{1}{2}(1+\psi(1-\lambda)) = -\frac{1}{2} - \psi(1-\lambda)$$

This function is negative for all values $\lambda \in [0,1]$ and $\psi \in [0,1]$, thus a marginal increase in both variables will correspond to a lower equilibrium profit for the Copyright Owner. As we know from the previous analysis that the two effects on the copyright owner have different signs when facing an marginal increase, we can deduce that the partial negative effect of λ on profits is stronger than the positive effect of ψ on profits when they both have a marginal increase.

We define *value effect* as the positive effect of a marginal increment in consumers' valuation in the alternative market good on the copyright owner's profits. This effect is quite clear, as an increment in consumers' valuation in the alternative market

leads to a higher price in the alternative market[15]. As the price in the alternative market is higher, the copyright owner will lower the price in the primary market to grant access at the secondary market to more consumers.

This model allowed us the opportunity to delve deeper in the mechanics of complementary products. Albeit this is a reduced and basic model, it allows us to define the three important interaction between the main variables in our model and the copyright owner's profits.

We define as "*interest effect*" the positive effect of a marginal decrease in the fraction of consumers interested in buying only the Information Good [16] on the copyright owner's profits[17].

Once again, this concept is quite intuitive: an increase in the fraction of consumers interested in buying only the Information Good implies a reduction of demand in the secondary market[18] (or vice versa, a decrease in the fraction of consumers interested in buying the bundle of the Information Good and the Alternative Good implies a decrease of the demand in the secondary market). As the demand is decreasing in the alternative market, the copyright owner would face a loss of profits. Thus, to compensate for the loss of profits, he will raise prices in the primary market.

We also study the cross-effect of a marginal increase in both the consumers' valuation for the Alternative Good ψ and the fraction of consumers interested only in the original good λ_0.

Last but not least, we define the multiplicative positive effect which stems from simultaneous marginal increase in consumers' valuation for the Alternative Good ψ and marginal decrease in the fraction of consumers interested in buying only the Information Good λ_0 as *virality effect*.

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[15] as shown in (8), $p_A^* = \psi$

[16] Notice that a decrease in the fraction in consumers interested in buying only the Information Good directly translates in an increase in the fraction of consumers interested in buying the bundle of the Information Good and the Alternative Good $\lambda_b = 1 - \lambda$.

[17] On the other hand, whenever the fraction of consumers interested in buying only the Information Good $\lambda_0 = \lambda$ increases, then the price in the primary market will increase and the monopolist profits will decrease.

[18] From (2.7):$D_A = (1 - \lambda)(1 - \hat{\theta})$

From the cross derivative it is clear that whenever the two variables have different signs, i.e. when ψ increase and λ decrease, there will be at the same time an increase in demand and in price in the alternative market. As the relationship between an increase in price and in demand is multiplicative and not additive, the profits will also increase in a multiplicative way. We decided to define it as "virality effect" as the requirement for it to exists is that there need to be a coordinated and coherent increase in consumers' valuation for the Alternative Good and an increase in consumers interested in buying the bundle of the Information Good and of the Alternative Good, thus whenever there is a high value effect of the alternative market and a high interest effect in the alternative market which can be translated in mundane terms as "going viral".

Those results can be summarized as follows.

In the Benchmark Model:

- A marginal increase in the consumer valuation for the Alternative Good ψ leads to a decrease in the equilibrium price in the primary market ($p_I^\star$) and to an increase in the monopolist equilibrium profits $\pi^\star$.

- A marginal decrease in the fraction of consumers interested only in the Information Good λ_0 leads to a decrease in the equilibrium price in the primary market ($p_I^\star$) and to an increase in the monopolist profits $\pi^\star$.

- An identical marginal increase in both the consumer valuation for the Alternative Good ψ and for the fraction of consumers interested only in the Information Good λ_0 leads to a decrease in the copyright owner's equilibrium profits $\pi^\star$. This implies that the negative effect of a marginal increase in the fraction of consumers interested only in the Information Good λ_0 on the copyright owner's profits is stronger than the effect ofthe positive effect that a marginal increase in consumers' valuation for the Alternative Good (ψ) on said profits.

It is worth to notice that the relationship between the effect of consumers' valuation for the Alternative Good ψ and the fraction of consumers interested only in the Information Good λ_0, is multiplicative.

Also, when studying the interaction between an Information Good and a complementary Alternative Good, we define:

- *Value effect* as the positive effect that an increase in consumers' valuation for the Alternative Good (ψ) has on copyroght owner's equilibrium profits.
- *Interest effect* as the positive effect that a decrease in the fraction of consumers interested only in consuming the Information Good (λ) has on copyroght owner's equilibrium profits.
- *Virality effect* as the multiplicative cross-effect that a contemporary increase in consumers' valuation for the Alternative Good (ψ) and a decrease the fraction of consumers interested only in consuming the Information Good (λ) has on copyroght owner's equilibrium profits.

Also, notice that the combination of those three effects express the intensity of complementarity in the model.

2.4.4 Consumers' consumption choice with End-User piracy

In a similar way to what we analyzed in the benchmark model, solving per backward induction, in time 2 consumers' will make their consumption choice. Taking into account the consumers' strategies $\{\theta, \lambda\}$ in the end-user digital piracy model, we can write the demand function for the Information Good and the secondary good given the consumers' utility functions as follow:

$$D_{\{\theta,\lambda\}} = \begin{cases} D_{\{b,0\}} \equiv \lambda(1-\tilde{\theta}) & \text{generated by consumers type } \{\theta_b, \lambda_0\}, \text{ for the Information Good.} \\ D_{\{p,b\}} \equiv (1-\lambda)(\tilde{\theta}-\hat{\theta}) & \text{generated by consumers type } \{\theta_p, \lambda_b\}, \text{ for the Alternative Good.} \\ D_{\{b,b\}} \equiv (1-\lambda)(1-\tilde{\theta}) & \text{generated by consumers type } \{\theta_b, \lambda_b\}, \text{ for both goods.} \end{cases}$$

Thus, the monopolist will face, for good I, demand:

$$D_I = (1 - \tilde{\theta}) \tag{2.13}$$

While the Copyright Owner will face, for good A, demand:

$$D_A = (1 - \lambda)(1 - \tilde{\theta}) + (1 - \lambda)(\tilde{\theta} - \hat{\theta})$$

Simplifying, we have that:

$$D_A = (1 - \lambda)(1 - \hat{\theta}) \tag{2.14}$$

We can summarize this result as follows. In time 2, for the indirect appropriation across complementary product model, the consumers' consumption choice generates:

- For good I, demand: $D_I = (1 - \tilde{\theta})$.
- For good A, demand: $D_A = (1 - \lambda)(1 - \hat{\theta})$

2.4.5 Copyright Owner's optimal pricing strategy in the Indirecta appropriation model

Lemma 2 *Solving per backward induction, in time 1, given the consumers' consumption choice stated previously:*

- *the equilibrium price for the Alternative Good s.t. the Copyright Owners profits are maximized is:* $p_A^* = \psi$.
- *the equilibrium price for the Information Good s.t. the Copyright Owners profits are maximized is:* $p_I^* = \frac{1}{2}(1 + M)(1 - \alpha)$.

Thus, the equilibrium profits given the equilibrium price for the Information Good and the Alternative Good are: $\pi^* = (\frac{1}{4}(M+1)(1+M+4\psi(1-\lambda)))\alpha - \frac{1}{4}(5+M(M+$

$2))\alpha^2 - M\psi(1-\lambda)$

From Subsection 3.2, we recall Equation (2.4), which express the copyright owner's monopolistic problem for the End-User digital piracy model:

$$\max_{p_I, p_A, \alpha} \pi = D_I p_I + D_A p_A - C_I - C_A - C_\alpha$$

Without loss of generality, we will set $C_I = 0$ and $C_A = 0$ for simplicity's sake.
We consider that in period two the Copyright Owner maximizes only profits based on the prices in both markets. Considering the demand shown in (2.13) and (2.14) we can rewrite (2.4) as follows:

$$\max_{p_I, p_A} \pi = (1 - \tilde{\theta})p_I + (1-\lambda)(1-\hat{\theta})p_A - \alpha \tag{2.15}$$

From Section 3.1 we recall (2.1) and (2.2):

$$\tilde{\theta} = \frac{p_I}{1-\alpha} - M \qquad\qquad \hat{\theta} = M(-1 + \frac{1}{\alpha})$$

From Subsection 3 we recall that $C_\alpha = \alpha$.
As the valuation of the secondary good ψ is common knowledge for the Copyright Owner, the CO will be able to set the maximum price when $V = 0$. As we know $V = \psi - p_A$, the Copyright Owner maximizes profits when $\psi_n - p_A = 0$. Thus, we find the equilibrium price for the alternative market is:

$$p_A^* = \psi \tag{2.16}$$

We can substitute all the results recalled or obtained in this Subsection into (2.14) and, through a few algebraic passages, we can re-write the Copyright Owner's profit

function as follows:

$$\max_{p_I} \pi = (1+M)p_I + \frac{p_I^2}{\alpha - 1} - \frac{(M(1-\alpha)-\alpha)(1-\lambda)\psi}{\alpha} - \alpha \tag{2.17}$$

As the first-order condition for profit maximization require $\frac{\partial \pi}{\partial p_I} = 0$, we take the first derivative of the profit function, obtaining:

$$\frac{\partial \pi}{\partial p_I} = 1 + M + \frac{2p_I}{\alpha - 1} \tag{2.18}$$

Setting (2.18) equal to zero and then solving for p_I, we find the equilibrium monopolistic price for the Information Good:

$$p_I^* = \frac{1}{2}(1+M)(1-\alpha) \tag{2.19}$$

Given those results, we can rewrite the profit function given the equilibrium prices as:

$$\pi^* = (\frac{1}{4}(M+1)(1+M+4\psi(1-\lambda)))\alpha - \frac{1}{4}(5+M(M+2))\alpha^2 - M\psi(1-\lambda) \tag{2.20}$$

2.4.6 Copyright Owner's optimal digital piracy level

Proposition 1 *Solving for backward induction, in the indirect appropriation across complementary goods model, at time 0 there exists a positive level of digital piracy in the market s.t. the monopolist profits are maximized, such level being:* $\alpha^* = \frac{2\sqrt{M\psi(1-\lambda)}}{\sqrt{5+M(2+M)}}$.

Note that as long as digital piracy exists in the market, α^* is an internal solution of the model. Thus, as long as digital piracy is allowed in the market it also exists an equilibrium level of digital piracy s.t. the copyright owner's monopolist profits are maximized.

Lemma 3 *Given the equilibrium level of digital piracy in the market, in time 0:*

- *the equilibrium price for the Information Good s.t. the Copyright Owners profits are maximized is:* $p_I^{**} = \frac{(1+M)(\sqrt{5+M(2+M)}-2\sqrt{M\psi(1-\lambda)})}{2\sqrt{5+M(2+M)}}$.

- *the equilibrium profits given the equilibrium price for the Information Good and the Alternative Good are:*
  $\pi^{**} = \frac{1}{4}+\frac{1}{4}M(M+2)+\psi+(M(1-\lambda)+\lambda)\psi-\sqrt{5+M(M+2)}\sqrt{M(1-\lambda)\psi}$

In the first period, the Copyright Owner will set the equilibrium level of digital piracy in the market such that profits are maximised given the equilibrium prices for the Information Good (p_I^*) shown in (2.19) and the Alternative Good (p_A^*) shown in (2.16). Given those values, the monopolist problem (2.15) can be rewritten for period three as:

$$\max_{p_I^*,p_A^*,\alpha} \pi = (1-\tilde{\theta})p_I^* + (1-\lambda)(1-\hat{\theta})p_A^* - \alpha \tag{2.21}$$

Recalling and adapting from the previous sections (2.1) and (2.2):

$$\tilde{\theta} = \frac{p_I^*}{1-\alpha} - M \qquad\qquad \hat{\theta} = M(-1+\frac{1}{\alpha})$$

and (16) and (19):

$$p_I^* = \frac{1}{2}(1+M)(1-\alpha) \qquad\qquad p_A^* = \psi$$

We can rewrite the monopolist problem for period three (2.16) as follows:

$$\max_{\alpha} \pi = \frac{1}{4}(1+M)(1+M+4\psi(1-\lambda)) - \frac{1}{\alpha}(M\psi(1-\lambda)) - \frac{1}{4}\alpha(5+M(2+M)) \tag{2.22}$$

The first order condition for this maximization problem is $\frac{\partial\pi}{\partial\alpha} = 0$. Taking the first

order derivative of the profit we thus obtain:

$$\frac{\partial \pi}{\partial \alpha} = \frac{1}{4}\alpha^2(-5-M(2+M)-\alpha(4(1+M)(\lambda-1)\psi)+4(M(\alpha-1)+\alpha)(\lambda-1)\psi) \quad (2.23)$$

Setting (2.23) equal to zero and solving for the piracy level α, we obtain two possible solutions:

$$\alpha = -\frac{2\sqrt{M\psi(1-\lambda)}}{\sqrt{5+M(2+M)}} \qquad \alpha = \frac{2\sqrt{M\psi(1-\lambda)}}{\sqrt{5+M(2+M)}}$$

As by assumption, the value α is included in the interval [0,1], the only acceptable solution for the equilibrium level of piracy yield:

$$\alpha^* = \frac{2\sqrt{M\psi(1-\lambda)}}{\sqrt{5+M(2+M)}} \quad (2.24)$$

Notice that this value α^* is positive for all fine $M > 0$, for all consumers' valuation for the Alternative Good $\psi \in (0,1)$ and for all fractions of consumers interested in consuming only the Information Good $\lambda \in (0,1)$. This implies that as long as digital piracy is allowed in the market, there is an internal solution for an equilibrium level of digital piracy in the primary market that allows for copyright owner's profit maximization.

This result also implies that in this theoretical framework exists indirect appropriation across complementary products as long as digital piracy exists in the market. Moving forward, given the equilibrium α^* shown in (2.24), we can rewrite the equilibrium price for the Information Good p_I^* shown in (2.16) as follows:

$$p_I^{**} = \frac{(1+M)(\sqrt{5+M(2+M)}-2\sqrt{M\psi(1-\lambda)})}{2\sqrt{5+M(2+M)}} \quad (2.25)$$

Given (2.16), (2.24) and (2.25) we can rewrite (2.20) as:

$$\pi^{**} = \frac{1}{4}+\frac{1}{4}M(M+2)+\psi+(M(1-\lambda)+\lambda)\psi-\sqrt{5+M(M+2)}\sqrt{M(1-\lambda)\psi} \quad (2.26)$$

2.4.7 Existence of Piracy

As noted in Subsection 3.1, for piracy to be allowed in the market we need the following condition to be verified: $\tilde{\theta} > \hat{\theta}$. From equations (2.1) and (2.2), we know that:

$$\frac{p_I}{1-\alpha} - M > M(\frac{1}{\alpha} - 1)$$

Through a few algebraic passages, it is easy to find that the condition to allow piracy in the market is:

$$p_I > \frac{(1-\alpha)M}{\alpha} \quad (2.27)$$

If we substitute the equilibrium level of piracy α^* shown in (2.23) into our condition, we can rewrite the Condition (2.27) as:

$$p_I > \frac{1}{2}M(\frac{\sqrt{5+M(2+M)}}{\sqrt{M\psi(1-\lambda)}} - 2)$$

We define $\underline{p}_I$ as the lower price that allows for piracy:

$$\underline{p}_I \equiv \frac{1}{2}M(\frac{\sqrt{5+M(2+M)}}{\sqrt{M\psi(1-\lambda)}} - 2) \quad (2.28)$$

Thus, for piracy to exist in the market, we have to verify the condition:

$$p_I^{**} > \underline{p}_I \quad (2.29)$$

Recalling (2.24) and (2.27), we can rewrite the Condition (2.28) as:

$$\frac{1}{2}(M+1)(1-\frac{2\sqrt{M\psi(1-\lambda)}}{\sqrt{5+M(2+M)}}) > \frac{1}{2}M(\frac{\sqrt{5+M(2+M)}}{\sqrt{M\psi(1-\lambda)}}-2)$$

The condition shown above is satisfied for fine ($M \in (0,\bar{M})$), for fraction of consumers interested only in consuming the Information Good ($\lambda \in (0,\bar{\lambda})$) and for consumers' valuation for the Alternative Good ($\psi \in \underline{\psi}, 1$).

Where: $\bar{M} = 0.295598$; $\underline{\psi} \equiv \frac{M(5+M(2+M))}{(1+M)^2}$; and $\bar{\lambda} \equiv 1 - \frac{M(5+M(2+M))}{(1+M)^2\psi}$.

Also, notice that when the fine M approaches the threshold $\bar{M}$, the consumers' valuation in the alternative market threshold required for existence of the model increases and the threshold fraction of consumers willing to consume only the Information Good required instead decreases. Thus, when $M \to \bar{M}$ we have that $\underline{\psi} \to 1$ and thus $\bar{\lambda} \to 0$. On the other hand, whether $M \to 0$, $\underline{\psi} \to 0$ and $\bar{\lambda} \to 1$.

As we know that α^* is profit-maximizing for all fine ($M > 0$), for all fractions of consumers interested in consuming the Information Good ($\lambda \in (0,1)$) and for all consumers' valuation for the Alternative Good ($\psi \in (0,1)$), we can state that as long as this condition is satisfied, then it exists a internal solution for our model.

The first insight that yield from this analysis is that for digital piracy to be allowed in the market we require three conditions, the first being the fine must be lower than the threshold ($\bar{M}$).

In this model, the fine (M) is the sanction for consumers which recourred to digital piracy to illegally consume the good and is exogenous as it is decided by government or by specialized copyright enforcement agencies. As a sanction, the fine (M) can also be seen as a deterrent toward the behaviour which is digital piracy.

From the results shown above, when the fine (M) reaches the threshold $\bar{M}$ the deterring effect of the fine should allow for piracy to be excluded from the market, coherently with what we should expect based on literature. What we find puzzling

is that in the real world, notwithstanding the efforts of copyright owner's aggregation in agencies for IP protection and the continuous effort by law enforcement and legislators, digital piracy is still growing strong.

From literature, we can reduce our reasoning to two main possibile reasons why for this outcome. The first possible reason is coherent with what shown in Hill (2007): consumers have distorted perception of the fine. Under this hypothesis, the threshold $\bar{M}$ is impossible to be reached due to the lack of moral pressure on consumers and the problem will be unsolvable as long as there will be higher consumer sensibility to the problem. The second possible reason why this does not happen in the real word is to state that in the real world we have yet to reach the threshold level $\bar{M}$. In this case, we can elaborate from an economic point of view that the benefits consumers receive from recouring to digital piracy are higher than the deterring effect of the fine (M) and thus we may be unable to reach this threshold in the real world. In both interpretation, we can argue that the fine (M) can be seen as an inefficient deterrent for digital piracy.

As far as the threshold required for consumers' valuation in the alternative market and for the fraction of consumers only interested in the Information Good, we can combine them saying that they reppresent the requirement in terms of the alternative attractiveness for consumers.

From this conclusions, we can derive the following observations.

When digital piracy is allowed in the market:

- For fine ($M \in (0, \bar{M})$)
- for fraction of consumers interested only in consuming the Information Good ($\lambda \in (0, \bar{\lambda})$)
- for consumers' valuation for the Alternative Good ($\psi \in (\underline{\psi}, 1)$).

Where: $\bar{M} = 0.295598$; $\underline{\psi} \equiv \frac{M(5+M(2+M))}{(1+M)^2}$; and $\bar{\lambda} \equiv 1 - \frac{M(5+M(2+M))}{(1+M)^2\psi}$.

Due to the interaction between the constraint for high enough consumers' valuation in the Alternative Good and the constraint for low enough fraction of consumers interested only in the Information Good, we can state that for indirect appropriation across complementary products to be efficient we need a sizeable alternative market. Also, we can see that when the fine overcomes the threshold ($M > \bar{M}$), digital piracy does not exist in the digital market.

On the other hand, when digital piracy is present in a market where it is possible indirect appropriation across complementary products, copyright owners should prefer a less stringent IP protection as a lower fine M reduces the requirement in term of alternative market to allow for indirect appropriation across complementary products.

Thus, although the model shows how it is possible for digital piracy to be pushed out of the market if there is enough deterrent effect from the fine (M), real data suggests that such deterrent effect is not quite as effective as expected.

2.5 Comparative statics

The next step in our analysis is to study how the variables affect the equilibrium values we found in the previous Section, to allow for further understanding and discussion in Subsection 7. We will analyze in Subsection 5.1 the comparative statics on the equilibrium price in time 1, in Subsection 5.2 the comparative statics on the equilibrium level of piracy in time 0, in Subsection 5.3 we will study the comparative statics on the equilibrium price given the equilibrium level of piracy. Last but not least, in Subsection 5.4 we will study the comparative statics on the equilibrium monopolist profits in time 0, π^{**}.

2.5.1 Comparative static on the equilibrium price in the primary market in time 1

In Subsection 4.5 we found the optimal pricing strategy for the monopolist given the consumers' choice of consumers. We recall from (2.19) the equilibrium price for the Information Good in the second period as follows:

$$p_I^* = \frac{1}{2}(1+M)(1-\alpha)$$

The two variables that affect the price on this stage are easily identified as the piracy level in the primary market (α) and the fine (M). To analyze how a marginal change in those variables affects the equilibrium price, we take the first derivative with respect to the variable we want to analyze and study its sign.

First of all, let's study the effect of a marginal increase in fine (M) on the equilibrium price for the second period.

$$\frac{\partial p_I^*}{\partial M} = \frac{1-\alpha}{2} \tag{2.30}$$

As this function is positive for all $\alpha \in [0, \bar{M}]$, we can determine that a marginal increase in the fine (M) will translate into a marginal increase in the equilibrium price in the primary market at the second stage of the game. This result is coherent with the results obtained in literature on basic analysis of digital piracy: the fine increases the monopolistic power of the copyright owner, thus it leads to higher equilibrium prices.

We now study the effects of the level of piracy (α) on the equilibrium price for the Information Good in the second period (p_I^*).

$$\frac{\partial p_I^*}{\partial \alpha} = -\frac{1}{2}(M+1) \tag{2.31}$$

This function is negative for all $M \in [0, \bar{M}]$, thus a marginal increase in the level of piracy in the digital market (α) leads to a decrease in the equilibrium price for

the primary market in the second stage. Once again, this result is coherent with the results obtained in the literature on basic analysis of digital piracy: digital piracy reduces the monopolistic power of the copyright owner, thus it leads to lower equilibrium prices.

2.5.2 Comparative statics on the equilibrium level of digital piracy in the primary market

Moving forward with our analysis, we can study the equilibrium level of piracy which allows the copyright owner for profit maximization given the equilibrium price in the primary and alternative market and the consumers' consumption choice (α^*). We recall from Subsection 4.6 Equation (2.24):

$$\alpha^* = \frac{2\sqrt{M\psi(1-\lambda)}}{\sqrt{5+M(2+M)}}$$

From (2.24) it is evident that three variables may affect the equilibrium level of digital piracy in the primary market (α^*), them being the fine (M), the consumers valuation for the Alternative Good (ψ) and the fraction of consumers interested in buying the Information Good (λ).

Also, Notice that from Subsection 4.7 we know that for piracy to be allowed in the market we have to respect the constraint: $M \in [0, \bar{M}]$, $\psi \in [\underline{\psi}, 1]$ and $\lambda \in [0, \bar{\lambda}]$, we will thus consider each function sign under this constraint.

First, we will proceed to study the effect of the fine (M) on the equilibrium level of piracy in the primary market (α^*).

$$\frac{\partial \alpha^*}{\partial M} = \frac{(5-M^2)(1-\lambda)\psi}{(5+M(2+M))^{3/2}\sqrt{M\psi(1-\lambda)}} \tag{2.32}$$

Under the constraint shown in Subsection 4.7 for piracy existence, this function is always positive. Thus a marginal increase in the fine M will, *ceteris paribus*, leads

to an increase in the equilibrium level piracy in the primary market. This result is counterintuitive if we take into consideration the insights supplied by literature on basic analysis of digital piracy: we should expect the strengthening in law enforcement (in this case due to an increase in the fine (M)) to lead a decrease in the level of digital piracy in the market.

To understand this result we need to examine the mechanics of the model closely: an increase in the fine M would indeed lead to less consumers recurring to digital piracy in the primary market, as it would cause the cutoff $\hat{\theta}$ to shift to the right[19]. This shift would decrease the demand in the alternative market[20]. To compensate for the demand shift in the alternative market, the copyright owner can only allow for a higher level of piracy in the primary market maintaining the *ceteris paribus* condition.

The second variable that affects the equilibrium level of digital piracy in the primary market (α^*) is the fraction of consumers interested in consuming only the Information Good (λ).

$$\frac{\partial \alpha^*}{\partial \lambda} = -\frac{M\psi}{\sqrt{5 + M(2+M)}\sqrt{M\psi(1-\lambda)}} \tag{2.33}$$

Under the limitation as per Subsection 4.7, this function will always be negative, consequently a marginal increase in the consumers interested in consuming only the Information Good will lead, *ceteris paribus* to a decrease in the equilibrium level of piracy in the primary market. This result is expected: if the fraction of consumers interested in the primary market increase, the weight of copyright owners' profits from the primary market increases while the demand for the Alternative Good decreases[21]. The Copyright Owner thus has incentives to reduce the level of piracy

[19] as shown in (2.2), $\hat{\theta} = M(\frac{1}{\alpha} - 1)$, thus for higher fine (M) the cutoff $\hat{\theta}$ will shift to the right.

[20] As the demand for the Alternative Goods shown in (14) $D_A = (1-\lambda)(1-\hat{\theta})$

[21] an increase in the fraction of consumers interested in the primary market increase, there is a concurrent decrease in consumers interested in buying the bundle of the intellectual and Alternative Good

in the market.

The third variable that affects the equilibrium level of digital piracy in the primary market (α^*) is the consumers' valuation for the Alternative Good (ψ).

$$\frac{\partial \alpha^*}{\partial \psi} = \frac{\sqrt{M\psi(1-\lambda)}}{\sqrt{5+M\psi(2+M)}} \tag{2.34}$$

Under the constraint presented in Subsection 4.7, this function is always positive. A marginal increase in the consumers' valuation for the Alternative Good (ψ) *ceteris paribus* leads to a increment in the equilibrium level of piracy in the primary market. This result is also expected: an increase in the consumers' valuation for the Alternative Good (ψ), *ceteris paribus* will lead to higher prices in the alternative market, thus the Copyright Owner will have incentive to grant access to more potential consumers in the alternative market and will thus tolerate higher digital piracy in the primary market.

2.5.3 Comparative statics on the equilibrium prices for the primary market in time 0

We will now analyze which variables affect the equilibrium price in the primary market given the equilibrium level of piracy in the primary market (p_I^{**}) and how they do so. We recall from 4.6 Equation (2.25):

$$p_I^{**} = \frac{(1+M)(\sqrt{5+M(2+M)} - 2\sqrt{M\psi(1-\lambda)})}{2\sqrt{5+M(2+M)}}$$

It is easily seen that the three variables that affect the equilibrium price in the primary market given the equilibrium level of piracy in the primary market (p_I^{**}) are the fine (M), the consumers' valuation for the Alternative Good (ψ) and the fraction of consumers interested in buying only the Information Good (λ).

As in the previous cases, the first variable we will study the effect on the equilibrium

price in the primary market given the equilibrium level of piracy in the primary market (p_I^{**}) is the fine (M).

$$\frac{\partial p_I^{**}}{\partial M} = \frac{1}{2}(1 - \frac{\psi(1-\lambda)(5+M(15+M(3M))}{(5+M(2+M))^{\frac{3}{2}}\sqrt{M\psi(1-\lambda)}}) \tag{2.35}$$

Under the assumption as per Subsection 4.7, this function is always negative. Thus, a marginal increase in fine (M) will lead *ceteris paribus* to a decrement in equilibrium price for the Information Good (p_I^{**}). This result overthrows what we showed in the comparative statics on prices at time 2, and it may appear counterintuitive and puzzling under the basic analysis of digital piracy results: an increase in fine should reduce digital piracy and strengthen the monopolistic power of copyright owners, thus leading to higher prices. But once again looking more closely at the hidden mechanics of the model reveals the reason behind this puzzling result. Similarly to what we saw previously, an increase in fine (M) led to a right shift of the cutoff $\hat{\theta}$, leading to a decrease in the demand for the Alternative Good. Once again, the copyright owner needs to compensate the decreasing demand in the alternative market, this time instead of allowing for more piracy in the primary market, the price will be lowered.

The second variable we study is the fraction of consumers interested in consuming only the Information Good (λ).

$$\frac{\partial p_I^{**}}{\partial \lambda} = \frac{M\psi(1+M)}{2\sqrt{5+M(2+M)}\sqrt{M\psi(1-\lambda)}} \tag{2.36}$$

Under the assumption as per Subsection 4.7, this function is always positive. Thus, a marginal increase in the fraction of consumers interested only in consuming the Information Good (λ) leads to an increase in the equilibrium price in the primary market given the equilibrium level of piracy in the primary market (p_I^{**}). This result is coherent with what shown in the benchmark model. A marginal increase in the fraction of consumers interested in consuming only the Information Goods λ implies

an incentive for the copyright owner to increase prices in the primary market.

Moving forward in our analysis, we will now analyze the effect of consumers' valuation from the alternative market ψ on the equilibrium price level in the primary market p_I^{**}.

$$\frac{\partial p_I^{**}}{\partial \psi} = -\frac{(1+M)\sqrt{M\psi(1-\lambda)}}{2\psi\sqrt{5+M(2+M)}} \tag{2.37}$$

Under the assumption as per Subsection 4.7, this function is always negative. Thus, a marginal increase in consumers' valuation for the Alternative Good will lead to a *ceteris paribus* decrease in the equilibrium price in the primary market. Once again, this result is coherent with what shown with regards to the benchmark model. Once again, this result is quite intuitive: an increase in consumers valuation for the alternative market is a direct incentive for the copyright owner to allow for more consumers to enter in the alternative market, and as we saw above one possible instrument to allow for more access in the alternative market is lowering the price in the primary market.

It is important to underline that while from academic literature on basic analysis of digital piracy we should expect an increase in fine (M) to lead to an increase in market power for the monopolists and thus to a higher equilibrium price in the primary market. While this holds true at a superficial level, once we reach the equilibrium level of digital piracy in the market the effect is the opposite: the right shift of the cutoff $\hat{\theta}$ would lead to a decrease in demand for the Alternative Good. The copyright owner can compensate for this effect by reducing the price in the primary market.

2.5.4 Comparative statics on the equilibrium copyright owner's profits

We move forward in our static analysis evaluating the equilibrium level of profits for the copyright owner, given the equilibrium level of piracy in the market π^{**}. We

recall from 4.6 Equation (2.26):

$$\pi^{**} = \frac{1}{4} + \frac{1}{4}M(M+2) + \psi + (M(1-\lambda)+\lambda)\psi - \sqrt{5+M(M+2)}\sqrt{M\psi(1-\lambda)}$$

The variables that affect the equilibrium level of profits given the equilibrium level of piracy in the market, given the equilibrium level of digital piracy in the primary market π^{**} are the fine M, the fraction of consumers interested in consuming only the Primary Good and the consumers' valuation for the alternative market ψ.
We start this analysis with the effects of fine M on the equilibrium copyright owner's profits, given the optimal level of piracy in the market.

$$\frac{\partial \pi^{**}}{\partial M} = \frac{1}{2}(1+M+\frac{\sqrt{M\psi(1-\lambda)}\,(2\sqrt{5+M(2+M)}\,\sqrt{M\psi(1-\lambda)}-5-M(4+3M))}{M\sqrt{5+M(2+M)}}) \tag{2.38}$$

Under the assumption as per Subsection 4.7, this function is always negative. This implies that a marginal increase in the fine (M) will translate to a decrease in the copyright owner's profits in equilibrium. Once again, based on the notions from the literature on basic analysis of digital piracy this result is counter-intuitive and conflicts with what we showed before: an increase in the fine (M) should translate in stronger monopolistic power for the copyright owner, and it thus should lead to higher profits. Once again, as we delve in the model mechanics, we see that the increase in fine (M) would lead to a rightward shift of the cutoff $\hat{\theta}$, thus leading to a decrease in the demand for the Alternative Good which leads to a loss in profits. This result strengthens the insights we gained in the previous subsection, as we have confirmation that an increase in fine (M) would lead to less profits, and the copyright owner is forced to allow for more digital piracy in the market or to lower the price in the primary market to keep profits stable.
We now proceed to analyze the marginal effect of the fraction of consumers interested in consuming only the Information Good λ on the equilibrium level of profits

given the equilibrium level of digital piracy in the primary market π^{**}.

$$\frac{\partial p_I^{**}}{\partial \lambda} = \frac{1}{2}\psi\left(\frac{5\sqrt{5+M(2+M)}}{\sqrt{M\psi(1-\lambda)}} - 2M - 2\right) \tag{2.39}$$

Under the assumption as per Subsection 4.7, this function is always negative. Thus a marginal increase in the fraction of consumers interested only in consuming the Information Good (λ) will lead to a decrease in the copyright owner equilibrium profits. This result is coherent with the mechanics shown in the benchmark model. Whenever the fraction of consumers interested only in the Primary Good increases, the demand for the Alternative Good will decrease and thus this will lead to a reduction in the copyright owner profits. Also, coherently with what shown before, the "interest effect" is still present in the model for indirect appropriation.

We proceed to evaluate the impact of a marginal increase in the consumers' valuation for the Alternative Good (ψ) on the equilibrium level of profits given the equilibrium level of digital piracy in the primary market π^{**}.

$$\frac{\partial \pi^{**}}{\partial \psi} = 1 + M - (1+M)\lambda - \frac{\sqrt{5+M(2+M)}\,\sqrt{M\psi(1-\lambda)}}{2\psi} \tag{2.40}$$

Under the assumptions as per Subsection 4.7, this function is always positive. Thus, a marginal increase in the consumers' valuation for the alternative market ψ will reflect an increase in the equilibrium level of profits in our model. Once again, this result is quite straightforward, as it is natural that an increase in consumers' valuation for one of the goods will lead to higher profits. Notice that also in this case, the "value effect" is still present in the market.

Last but not least, we investigate for the cross-effect given by a marginal increase in both the consumers' valuation for an Alternative Good (ψ) and of the fraction of consumers interested only in consuming the Information Good (λ). Notice that as both ψ and λ are continuous variables, we can apply the Schwarz Theorem and

thus we have symmetry of the second derivatives, thus $\frac{\partial^2 \pi^{**}}{\partial \lambda \partial \psi} = \frac{\partial^2 \pi^{**}}{\partial \psi \partial \lambda}$.

$$\frac{\partial^2 \pi^{**}}{\partial \lambda \partial \psi} = -1 - M + \frac{M\sqrt{5 + M(2 + M)}}{4\sqrt{M\psi(1 - \lambda)}} \tag{2.41}$$

Under the constraint as per Subsection 4.7, this function is always negative. Thus, when both the consumers' valuation for the Alternative Good (ψ) and the fraction of consumers interested only in the Primary Good (λ) increase, the monopolist profits will decrease. Once again, this result shows that when digital piracy is present in the market the negative effect of the marginal increase in the fraction of consumers interested only in consuming the Information Good is also stronger than the positive effect due to an increase in consumers' valuation for the Alternative Good.

In Chapter 1.5 we defined that indirect appropriation arises in a market where illegal copies are produced whereas the copyright owner is able to capture part or all the value lost because of said copies. Given this definition, we can state that value effect, interest effect and virality effect reppresent the positive effects of complementary markets in the model, can also be seen as proxy of the strenght of indirect appropriation across complementary products.

Once again, from academic literature on basic analysis of digital piracy we should expect an increase in fine (M) to lead to an increase in market power for the monopolists and thus in higher equilibrium profits. While this holds true when digital piracy is still exogenous, once we take into account the equilibrium level of digital piracy the results is overthrown: the loss of demand in the alternative market generated by the increase in law enforcement (in this case through the fine (M)) leads to a reduction of the copyright owner's profits.

2.6 Value effects, Interest effects and Virality effects with and without digital piracy

In the previous section we introduced as *value effect* the positive effect that an increase in consumers' valuation for the Alternative Good (ψ) has on copyright owner's equilibrium profits, as *interest effect* the positive effect that a decrease in consumers interested only in consuming the Information Good brings on copyright owner's profits and as *virality effect* the positive multiplicative cross-effect birthed by a contestual increase in consumers' valuation for the Alternative Good (ψ) and decrease in fraction of consumers interested in consuming only the Alternative Good (λ). In Subsection 4.3 we defined the value effect, the interest effect and the virality effect for both the benchmark model and in Subsection 5.4 we evaluated their values for the indirect appropriation model. Thus, it is possible to make a comparison between the two variables to see if and how the presence of digital piracy affects the intensity of those effect.

Before studying the comparate strength of value and interest effect in the two models, we need to introduce the threshold:

$\ddot{M} = \frac{-11+11\lambda(2-\lambda)(2-\lambda(2-\lambda))+(-10-3\sqrt{159}\sqrt{(1-\lambda)^{12}}+10\lambda(2-\lambda)(3-\lambda(3-\lambda))(1-\lambda(1-\lambda)))^{\frac{2}{3}}}{2(\lambda-1)^3(-10-3\sqrt{159}\sqrt{(1-\lambda)^{12}}+10\lambda(2-\lambda)(3-\lambda(3-\lambda))(1-\lambda(1-\lambda)))^{\frac{1}{3}}}$

Value effect and interest effect are stronger in the model with digital piracy for $M \in (0, \ddot{M})$ and will be instead weaker for $M \in (\ddot{M}, \bar{M})$. Also, value effect and interest effect will have the same strenght in both models and for $M \in (\ddot{M}, \bar{M})$.

To study virality effect we require additional thresholds on the consumers' valuation for the Alternative Good ψ, on the fraction of consumers interested only in consuming the Information Good (λ) and on the fine (M).

To simplify the exposition, we will introduce the following notation in supplement to the one introduced above:

$\dot{\psi} = \frac{-11+11\lambda(2-\lambda)(2-\lambda(2-\lambda))+(-10-3\sqrt{159}\sqrt{(1-\lambda)^{12}}+10\lambda(2-\lambda)(3-\lambda(3-\lambda))(1-\lambda(1-\lambda)))^{\frac{2}{3}}}{2(\lambda-1)^3(-10-3\sqrt{159}\sqrt{(1-\lambda)^{12}}+10\lambda(2-\lambda)(3-\lambda(3-\lambda))(1-\lambda(1-\lambda)))^{\frac{1}{3}}},$

$\ddot{\psi} = \frac{1}{2-2\lambda}$

$\dot{\lambda} = 0.023571$

$\ddot{\lambda} = 0.700235$

It also exists a threshold $\dot{M}$, which is displayed in appendix due to space constraints. Notice that those boundaries are all comprised between 0 and 1 and they all are allowed for the existancy conditions of digital piracy as per Subsection 4.7.

Given the aforementioned notation, virality effect is weaker when digital piracy is present in the market:

- for fraction of consumer interested only in the Primary Good $0 < \lambda \leq \dot{\lambda}$, for consumers' valuation for the Alternative Good $\dot{\psi} < \psi < \ddot{\psi}$ and for fine $\dot{M} < M < \ddot{M}$
- for fraction of consumer interested only in the Primary Good $0 < \lambda < \dot{\lambda}$, for consumers' valuation for the Alternative Good $\ddot{\psi} < \psi < 1$ and for fine $0 < M < \ddot{M}$
- for fraction of consumer interested only in the Primary Good $\dot{\lambda} < \lambda \leq \frac{1}{2}$, for consumers' valuation for the Alternative Good $\dot{\psi} < \psi < \ddot{\psi}$ and for fine $\dot{M} < M < \ddot{M}$
- for fraction of consumer interested only in the Primary Good $\dot{\lambda} < \lambda \leq \frac{1}{2}$, for consumers' valuation for the Alternative Good $\ddot{\psi} < \psi < 1$ and for fine $0 < M < \ddot{M}$
- for fraction of consumer interested only in the Primary Good $\frac{1}{2} < \lambda < \ddot{\lambda}$, for consumers' valuation for the Alternative Good $\ddot{\psi} < \psi < 1$ and for fine $\dot{M} < M < \ddot{M}$

Virality effect will be stronger in the market when digital piracy is present in the market:

- for fraction of consumer interested only in the Primary Good $0 < \lambda \leq \frac{1}{2}$, for consumers' valuation for the Alternative Good $0 < \psi < \dot{\psi}$ and for fine

$0 < M < \dot{M}$,

- for fraction of consumer interested only in the Primary Good $0 < \lambda \leq \frac{1}{2}$, for consumers' valuation for the Alternative Good $\dot{\psi} < \psi < \ddot{\psi}$ and for fine $\dot{M} < M < \ddot{M}$
- for fraction of consumer interested only in the Primary Good $\frac{1}{2} < \lambda < 1$, for consumers' valuation for the Alternative Good $0 < \psi < \ddot{\psi}$ and for fine $0 < M < \dot{M}$
- for fraction of consumer interested only in the Primary Good $\frac{1}{2} < \lambda < 1$, for consumers' valuation for the Alternative Good $\dot{\psi} < \psi < 1$ and for fine $\dot{M} < M < \ddot{M}$
- for fraction of consumer interested only in the Primary Good $\frac{1}{2} < \lambda < \ddot{\lambda}$, for consumers' valuation for the Alternative Good $0 < \psi < 1$ and for fine $0 < M < \dot{M}$

The intensity of virality effect will instead be equal in both the model with and without digital piracy:

- for fraction of consumer interested only in the Primary Good $0 < \lambda \leq \frac{1}{2}$, for consumers' valuation for the Alternative Good $\dot{\psi} < \psi < \ddot{\psi}$ and for fine $M = \dot{M}$
- for fraction of consumer interested only in the Primary Good $\frac{1}{2} < \lambda < 1$, for consumers' valuation for the Alternative Good $\ddot{\psi} < \psi < 1$ and for fine $M = \dot{M}$

It is important to cross-reference these results with the ones from Subsection 4.7. From the condition for piracy existance we learnt that indirect appropriation across consumers is possible when there is enough interest in the alternative market, i.e. when there is a high enough consumers' valuation for the Alternative Good and a low enough fraction of consumers interested in consuming only the Information

Good.

To clarify this result, we need to that we require a certain level of interest in alternative market for indirect appropriation to be possibile in the market, i.e. we require high enough consumers' valuation for the Alternative Good and a low enough fraction of consumers interested in consuming only the Information Good. Based on this, we can state that both value effect and interest effect intensity are stronger when the fine (M) is low ($M \in (0, \ddot{M})$), while they will be weaker for high fine (M) with ($M \in (\ddot{M}, \bar{M})$). *Ceteris paribus* thus an higher fine will lead to weaker complementarity positive effects in the market. This implies that copyright owners should prefer lower copyright enforcement in the form of the fine (M), to allow for stronger indirect appropriation across complementary products.

For the virality the situation may appear more complicated, but once we account for the condition aforementioned (a certain level of interest in alternative market for indirect appropriation to be possibile in the market), we can clearly see that once again the virality effect is stronger when in the market is allowed digital piracy for low fine ($M \in (0, \ddot{M})$) and is weaker for high fine ($M \in (\ddot{M}, \bar{M})$). Once again, copyright owners should prefer lower copyright enforcement in the form of the fine (M), to allow for stronger indirect appropriation across complementary products.

These insights can be summarised as follows.

When digital piracy is introduced in the market, the value effect and the interest effect will be stronger w.r.t the benchmark model for low fine ($M \in (0, \ddot{M})$) and will be weaker for high fine ($M \in (\ddot{M}, \bar{M})$). The effects will instead have the same strength in both models for ($M = \ddot{M}$).

When digital piracy is introduced in the market, the virality effect will be stronger w.r.t the benchmark model for:

- for $0 < \lambda \leq \frac{1}{2}$, for $0 < \psi < \dot{\psi}$ and for $0 < M < \dot{M}$,
- for $0 < \lambda \leq \frac{1}{2}$, for $\dot{\psi} < \psi < \ddot{\psi}$ and for $\dot{M} < M < \ddot{M}$

- for $\frac{1}{2} < \lambda < 1$, for $0 < \psi < \ddot{\psi}$ and for $0 < M < \dot{M}$
- for $\frac{1}{2} < \lambda < 1$, for $\dot{\psi} < \psi < 1$ and for $\dot{M} < M < \ddot{M}$
- for $\frac{1}{2} < \lambda < \ddot{\lambda}$, for $0 < \psi < 1$ and for $0 < M < \dot{M}$

Virality effect will be weaker w.r.t. the benchmark model:

- for $0 < \lambda \leq \dot{\lambda}$, for $\dot{\psi} < \psi < \ddot{\psi}$ and for $\dot{M} < M < \ddot{M}$
- for $0 < \lambda < \dot{\lambda}$, for $\ddot{\psi} < \psi < 1$ and for $0 < M < \ddot{M}$
- for $\dot{\lambda} < \lambda \leq \frac{1}{2}$, for $\dot{\psi} < \psi < \ddot{\psi}$ and for $\dot{M} < M < \ddot{M}$
- for $\dot{\lambda} < \lambda \leq \frac{1}{2}$, for $\ddot{\psi} < \psi < 1$ and for $0 < M < \ddot{M}$
- for $\frac{1}{2} < \lambda < \ddot{\lambda}$, for $\ddot{\psi} < \psi < 1$ and for $\dot{M} < M < \ddot{M}$

The effect instead will have same strength in both models for:

- for $0 < \lambda \leq \frac{1}{2}$, for $\dot{\psi} < \psi < \ddot{\psi}$ and for $M = \dot{M}$
- for $\frac{1}{2} < \lambda < 1$, for $\ddot{\psi} < \psi < 1$ and for $M = \dot{M}$

When digital piracy is introduced in the market, and under the general requirement for digital piracy existance (high enough consumers' valuation for the Alternative Good and a low enough fraction of consumers interested in consuming only the Information Good), complementarity between the primary market and the alternative market expressed in the form of "value effect", "interest effect" and "virality effect" will be stronger for low fine ($M \in (0, \ddot{M})$) and will be weaker for high fine ($M \in (\ddot{M}, \bar{M})$). The effects will instead have the same strength in both models for ($M = \ddot{M}$).

Also, it is important to notice that when digital piracy is present in the market, copyright owners should prefer low fine (M), as when the fine (M) is lower, indirect appropriation across complementary good is stronger.

2.7 Result discussion and policy's implications

Up until this section we avoided discussing the deeper implications of our findings. Now that we derived the full model, we can discuss those findings in a more organic and comprehensive way.

The first insight we gained in our analysis is based on the interaction mechanics of an Information Good with a complementary Alternative Goods. The insight, consists in the presence of three different effects on profits due to the interaction between primary and alternative market. We thus defined as *value effect* the positive effect that an increase in consumers' valuation for the Alternative Good (ψ) has on copyright owner's equilibrium profits, as *interest effect* the positive effect that a decrease in consumers interested only in consuming the Information Good brings on copyright owner's profits and as *virality effect* the positive multiplicative cross-effect birthed by a contestual increase in consumers' valuation for the Alternative Good (ψ) and decrease in fraction of consumers interested in consuming only the Alternative Good (λ). We also introduced how these effects are a proxy of the positive effect of complementarity in the model. The implication of this definition comes to fruition when we add digital piracy in the model, as those effects assume an identity as a proxy measure of indirect appropriation: the higher the positive effect of complementarity is in the market, the stronger indirect appropriation across complementary product will be. Thus it becomes of paramount importance for our model to determine what sign these effects assume when digital piracy is introduced in the market.

On this topic, we present a comparison in strenght between the benchmark model and the indirect appropriation model. From those results we see that digital piracy may strengthen or weaken all those effects, but we can notice that when there is high enough consumers' valuation for the Alternative Good and low enough fraction of consumers' interested only in consuming the Information Good in the market,

the factor which determines if those effects are stronger or weaker is the fine (M). Also we conclude that as when the fine (M) is higher, all those effects are weaker, copyright owner should prefer weaker IP right protection in the form of the fine (M) rather than a strong copyright protection as it would allow for higher degree of indirect appropriation in the market. This result is confirmed from the digital piracy existancy condition, as when the fine (M) is low then the restriction on the alternative market relaxes. The implications of those results are quite strong from a policy point of view, as they imply that copyright owner should press for lower fines in the market, instead that for higher fines. Albeit this type of result is already present in literature on digital piracy, it has been up untill now associated with literature on network effects and not to literature on indirect appropriation. Notwithstanding, the fact that we find the same result in two different branches of literature solidify and strenghten this result, in our opinion.

The second insight we gained in this model is the existance of an equilibrium level of digital piracy s.t. the copyright owner's profits are maximized in the market and that said solution is an internal solution as long as digital piracy is allowed in the market. This result is our main result as it shows that indeed indirect appropriation across complementary products could be reppresented in a theoretical model with endogenous digital piracy. The implications of this result are quite important, as it shows that digital piracy is not a menance to firms, but instead an opportunity.

To use digital piracy as an instrument we need a policy change that could allow willing authors and copyright owners to act. Thus, we think that from a policy perspective would be a sound approach if governments enact a legislation to fully integrate Copyright Owners into the mechanism of control and enforcement over digital piracy for their products, as it could become possible for big firms with alternative markets to reach profit maximization in spite of the presence of digital piracy. In such a scenario, firms could decide in concert with law enforcement agencies the level of the fine M to allow for an increase in their profits by increasing

or reducing the value as they see fit. This type of system could be fully integrated with a new distribution business model on freemium or premium views such as the ones shown in Aversa et al (2019).

Moving forward in our analysis, with these models we introduce the concept that a high enough fine should be able to deter consumers from recurring to digital piracy. This result is in a certain way similar to the one obtained by Banerjee (2006). In Banerjee (2006) the results suggest that it exists a certain threshold over which lobbying for copyright protection is the optimal solution in the digital market. As we already stated, although in the real world both governments and law enforcement agencies are not sparing any effort to reach such a result, the projection shown in literature [22] all seems to unanimously affirm the growth for digital piracy in future years. Also, the data show that just a few law enforcement interventions had a reflection on consumers. They are the ones against pay-to-view platforms: as those platforms engage in Commercial piracy, the subscriber can be found and persecuted by law. An example is what recently happened in Italy with the breakdown of the digital platform Xtream[23] where consumers risk fines and even jail time. On the other hand breakdown against p2p platforms such as MegaUpload did not lead to consumers being forced to pay fines, and the sign of the effect of such interventions on the primary market is still object of debates in economic literature[24] as it does not seem to scare that much digital pirates: consumers will just move to other platforms and try new channels to continue their behaviour[25].

There are multiple ways to try to solve this problem. As suggested in Danaher et al (2019) governments may try to encompass all channels whenever they shut down a platform. Or as shown in Banerjee (2006) lobbying may lead to less piracy entry in the market. Other possible ways to interact are applying new distribution models such as the one proposed by Spotify and Netflix. In our opinion a possible way

[22] 2017 Frontier Economics report for BASCAP and INTA
[23] Fonterosa (2019)
[24] see Peukert et al. (2017) and Danaher et al (2019) on this topic
[25] Danaher et al (2019)

to solve the problem should be to enact all the previous suggested methods, while governments and law enforcement agencies should enhance the efforts in enforcement focused against end-user piracy. This type of intervention should raise the perception of the fine in consumers, possibly in concert with a sensibilization social campaign on digital piracy in schools.

If a more comprehensive and flexible system of IP right enforcement is introduced, this would allow also to solve one of the main problems when designing a system to protect IP rights: copyright owners and artists are not always on the same page on how to enforce or react to digital piracy and even to ad-payd music in case of freemium models. A few examples: Radiohead and Taylor Swift both took down their music from Spotify, Lily Allen and James Blunt instead participated in a direct campaign against Öle-sharing. On the other hand of the spectrum the copyright owner Adami in France (as shown in Bacache-Beuvallet et al (2015)) directly stated that artists are just not willing to sue their fans[26] and artists such as Skrillex, Franz Ferdinand and Trent Reznor openly encouraged their fans to pirate their music. A customizable system such as the one we propose, where the copyright owner or artist is fully integrated into how to treat digital piracy and how harsh the punishment of digital pirate is, could allow for all those different opinions to fit and coexist peacefully, as each copyright owner or Artist could directly manifest his positions on the topic.

When taking into consideration the fine (M), up untill this point we already defined that when digital piracy is present in the market copyright owner's have incentive to ask for lower fines, instead of higher ones. This result is even more true when we take into consideration the insights derived on the relationship between the fine (M) and respectively the equilibrium price in the primary market, the equilibrium level of digital piracy in the primary market and the copyright owner's profits are quite relevant, as they overthrow some common concept in economic literature.

[26]source: http://www.adami.fr/defendre-les-droits-des-artistes.html.

While on a general approach those results may appear as counter-intuitive, the in depth analysis of the model mechanics allows for understanding the ratio behind our results: as a marginal increase in fine (M) leads to a decrease in copyright owner profits due to the reduction in demand for the Alternative Good, the copyright owner to keep the situation balanced may act one of two ways, he may allow for higher level of digital piracy in the primary market or he can decrease the prices in the primary market, as both actions would lead to counteract the negative effect of the fine (M) on the demand for the Alternative Good. Notice that this reasoning also allows us to understand the hidden mechanics: the positive effect of the fine (M) on the primary market due is weaker than the negative effect on the demand function for the Alternative Good. Notice that this result is quite similar to the results from Takeyama (1994): our results show that although on a general approach it may appear that strengthening enforcement against copyright allows copyright owner to strengthen their monopolistic power and counteract digital piracy, a more in depth analysis reveals that it is instead a profit-maximizing behaviour for the copyright owner to be tolerant toward digital piracy as it may lead to opportunities in neighbouring markets for complementary Alternative Good.

This idea supports once again our previous statement concerning the necessity of a deeper integration of copyright owners and artists in the control and enforcement of IPRs: while it is true and undeniable that piracy is dangerous and that it shifts profits from the intellectual market, in an environment where there are complementary Alternative Goods it may become an important instrument in the hand of firms to allow for profit maximization. Thus it is auspicable a system where the copyright owners are fully integrated into the decision on the strength of the punishment and on the strictness of the enforcement of said punishment: this could allow for each owner to freely decide the most fitting course of action for his/hers own benefits, taking into account their own idiosyncrasy. Also, Notice that a similar system would be compatible with the new business distribution models for the

digital market that are already growing in the market, and could even enhance their performances with more fitting enforcing profile for their own best interest.

2.8 Limits of the model, future development and concluding remarks

This model is far from being perfect or complete, as it still is a simplified model with numerous weaknesses that could and hopefully will be covered in future extensions. The first weakness and the first development direction will be supplementing the model with network effects. As we are dealing with complementary Information Goods that create a franchise aiming to create a full-immersive experience which involves multiple consumers, the inclusion in the model of network effects brought by world-to-mouth spreading must be considered, especially in an environment where the virtual "buzz" is a key element in obtaining widespread fame and success. The second avenue of research we wish to explore is the introduction of a welfare analysis, to derive stronger and more detailed policy implications. Other possible extensions include, relaxing the assumption on the dichotomous consumers' preference for the alternative market in favor of a continuous setting, relaxing the assumption on the original and the copy being of the same quality and also allowing for digital piracy in the alternative market.

Although the model has flaws and can be strengthened with those extensions, it is still based on a solid theoretical framework, the Mussa and Rosen vertical differentiation, which has been widely used in related literature on digital piracy. Notice that although in recent years economic research has shifted its focus toward the new distribution channels and toward the new business model arising from digitalization, indirect appropriation across products is still one of the main approaches for big firms and franchises. Although being clearly a widespread business strategy, this model is - at the best of our knowledge and at the time of our writing - the

only theoretical model on indirect appropriation with endogenous digital piracy. We hope it will become a foundation for more complete and strong theories in the near future.

On this spirit, in the following Chapter we will present an extension of this model in which we will evaluate the impact of network externalities in this framework for indirect appropriation for complementary Information Goods.

Chapter 3

Indirect appropriation for complementary Information Goods and Network Effects

Abstract

When a bundle of Information Goods and/or physical goods create a shared universe, consumers tend to for what is commonly known as "fandom". This type of aggregation phenomenon enhances the consumers' consumption experience and thus it is a modelled as a consumption externality well known in digital economy as Network Effects. As we saw in the previous Chapter, Information Goods are also plagued by the phenomenon known as Digital Piracy. In this work, we will extend our previous work in the field of indirect appropriation across complementary products to allow for buzz network effects, where buzz network effects are the positive within-group external effects due to consumption of the same good, either legally or illegally. This model follows an à la Mussa and Rosen (1975) approach for vertical differentiation, where an Information Good may or may not be consumed

with a complementary physical or information Alternative Good sold in an alternative market. Our results not only show the basic mechanics of interaction between indirect appropriation across complementary products and network externalities, but also show a theoretical proof of the inefficiency of the fine (M) as an instrument to drive digital piracy are present in a market with strong network effects.

3.1 Introduction

In the previous Chapter we presented a simple model for indirect appropriation across products, based on a Mussa and Rosen approach with endogenous digital piracy. When introducing our previous model, we took as a focus of our research idea industries which base their revenues on the ability to create a universe, almost a web, of products - intellectual or physical - interconnected on multiple levels to create consumption experience. Those industries include the movie industry, the physical game industry, the videogames industry, the music industry and the printed book industry, as well as the sport industry: the creation of complementary physical or Information Goods to enhance the consumption of a certain Information Good to allow for indirect appropriation is widespread and common, especially in the interconnected modern world.

Some of the concepts that comes to mind in this environment are "fandom", "getting hyped" and "going viral". While fandom is an "old" sociological phenomenon, based on creation of subcultural nucleus which shares common beliefs and interests toward certain elements of pop culture[1], digitalization has created a full new dimension of this phenomenon. Consider phenomena like D23 for Disney, Pottermore

[1]John Fiske in "The Adoring Audience: fan culture and popular media" define fandom as *a common feature of popular culture in industrial societies. It selects certain performers, narratives or genres and takes them into the culture of a self-selected fraction of the people. They are then reworked into an intensely pleasurable, intensely signifying popular culture that is both similar to, yet significantly different from, the culture of more 'normal' popular audiences [...] Fans create a fan culture with its own systems of production and distribution that forms what i shall call 'a shadow cultural economy' that lies outside that of the cultural industries yet shares features with them".*

for the Wizarding World of Harry Potter, Nintendo Direct for Nintendo, Sony Live Streams for Sony and so on. All these constructs generate hype around the companies through announcements of new products of their franchises, thus generating hype among consumers. Products publicised on such massive platforms will gain particular attention for the public and will "go viral" through the net, becoming widespread in a few hours across the globe. Like a stone thrown into a lake, when such a platform makes an announcement, ripples will form on the surface of the web: the whole machine of fans will activate through forums, YouTube content creators, specialized press and so on creating more and more ripples. Also, all those platforms allow users to interact to various degrees, spreading word to mouth experience of the various products available on the market from the franchise.

While it is true that sharing information may appear as a consumer sampling topic, the demand generated by world-to-mouth (in this case amplified by digital services) suggestions and aggregation are exactly the definition of what are called in literature Buzz Network Effects. But what are network effects?

Belleflamme (2016) shows various challenges facing new economists to proceed in the creation of a new model on the digital market given the recent changes. These changes are due to new players such as Spotify, Youtube and Netflix and the new distribution models they brought onto the market, such as ad-paid views, freemium or premium consumption. One of those challenges is given by the modelling of within-group external effects. Those effects, as suggested in Belleflamme (2016), are due to two main reasons: the benefits of interacting with other users (i.e. sharing playlist) and recommender systems (i.e. users discovery of high-value content). As we noted in the previous paragraph, this is exactly the type of behaviour we want to capture in our work.

While taking account of network effects, we can face two possible types of positive within-effects. In some cases the effect is due to the number of goods consumed by consumers. As an example of this type of within-network effect is Game of

Thrones: Jeff Bewkes, CEO of Time Warner, the company which owns HBO, said that *"I think you're right, that 'Game of Thrones' is the most pirated show in the world [...] Now, that's better than an Emmy"*[2], and other executives of HBO noted that the contribution of this "cultural buzz" to the series worldwide success. This type of phenomenon is not new to economic literature, as it has been also noted by Belleflamme and Peitz (2012) in regard to the first episode of the third season of the GoT show. This type of network effects involves anyone who consumes the good, either legally or illegally and we will refer to this type of network effect as *"cultural buzz network effect"* or in short *"buzz network effect"*.

The second type of positive within-effect instead occurs when the extra utility gained by users depends exclusively on the users who buy the legal copy. Examples of firms which use this type of network effect are Spotify, Anobii and Steam: browsing along your friends playlist you can find new music genres, songs, books or games which may match your tastes. Often in this type of system you can also share with others your wishes and create a wishlist and others may even buy the good for you. Those are only some of the new applications of this type of effect: moving back in time, a well known example of this type of network effect is the one we saw in Chapter 1.6 in the software industry for software compatibility. The study by Economides and Viard (2007) models this type of network effect for a complementary product.

While our study focus may allow for both these effects, the main focus of this work is the buzz network effect and our model will develop on this idea.

3.2 Related Literature

As this model is a direct extension of the model presented in Chapter 2, for the related literature on the model construction and theoretical framework we directly cross-reference Chapter 2.3. For general literature on digital piracy and network

[2]Caitlin Dewey -"Game of Thrones exec says piracy is better than Emmy. He has a point." - Washington Post and Caitlin

effect, instead, we reference to Chapter 1 and Section 1.6. Thus, in this Section, we focus only on the main studies which relate to the reduced form of Network Effects we introduce in our model.

The first contribution on this type of framework for Network Effects has been developed from Katz and Shapiro (1985), which derived the general setting which captures the network effects in the market. The framework was further developed by Farrell and Saloner (1986), who adapt it in a market with complementary goods. In this case, the authors presented a market for a new technology which may be compatible or incompatible with an already existing installed base. In more recent years Economides and Viard (2007) applied this type of network effects in a "mix and match" model for pricing strategies for complementary goods such as Microsoft OS and Office Suite.

In Chapter 2.3 we introduced Economides and Viard (2007), and while what we already discussed on the relationship between the previous model and this model still holds, when we take into account network effects we have to include the modelling of network effects. While Economides and Viard considers a network effect based on the number of sales of products and study 5 scenarios of complementary goods with network effects, in our model we present a model which is based on the buzz network effect and study the interaction between digital piracy, indirect appropriation and network effects.

Although in literature already exist some works which apply a Mussa and Rosen approach to network effect, such as Takeyama (1994), to the best of our knowledge, there is no theoretical model which considers complementary goods and thus creates a link between literature on indirect appropriation and literature on network effect has as yet been proposed with this type of setting.

3.3 The model of indirect appropriation with buzz network effects

Similarly as what we encountered in the the previous model, we will consider two markets: one for the Information Good (good I), also known as primary market, and one for an Alternative Good (good A), also known as an alternative market. In these markets the copyright owner due to the effect of copyright protection offers both goods as a monopolist, while consumers will generate a demand for both goods based on their utility functions.

In the primary market we present a Mussa and Rosen approach, where the monopolist (Copyright Owner) faces a continuum of potential users uniformly distributed in a segment of unitary dimension. Potential users are characterized by their valuation θ for the Information Good for $\theta \in [0, 1]$. In the primary market, consumers have three alternatives: they may choose to consume the good legally, thus buying the original at price (p_I), they may choose to consume an illegal copy of the original (thus becoming "digital pirates") or they can decide to not consume the good at all. When using digital piracy, consumers face a probability $(1-\alpha)$ of being caught in the act, with $\alpha \in [0, 1]$ being the chance of not being caught by law enforcement. In a similar way to the previous Chapter, based on the results in Yao (2005), we also assume α to be a proxy variable for the level of digital piracy present in the market. When consuming the good, either legally or illegally, consumers will derive an extra utility based on the number of consumers who consumed the good either legally or illegally (x) based on a certain level of intensity of network effects β for $\beta \in [0, 1]$, where (βx) represent the buzz network effect.

Given this settings, we can write the utility consumers derive in the primary market

as follows:

$$U = \begin{cases} \theta - p_I + \beta x & \text{if the consumer buys the digital good} \\ \alpha\theta - (1-\alpha)M + \beta x & \text{if the consumer consumes the digital good illegally} \\ 0 & \text{otherwise} \end{cases}$$

In the alternative market, once again, we consider a simple market supplied by a third party through a franchise agreement, where the Copyright Owner will receive a royalty p_A. Consumers in the alternative market have a dichotomous preference based on their valuation of the Alternative Good ψ for $\psi \in [0,1]$. For simplicity's sake we assume the consumers valuation ψ to be common knowledge for the copyright owner. Based on their preference on ψ, consumers' fraction $\lambda_0 = \lambda$ for $\lambda \in [0,1]$ will never be interested in the Alternative Good, while fraction $\lambda_b = 1 - \lambda$ of consumers may be interested in buying a bundle of the Information Good and the Alternative Good.

The utility consumers derive while consuming the Alternative Good is:

$$V = \begin{cases} \psi - p_A & \text{if the consumer buys the Alternative Good} \\ 0 & \text{otherwise} \end{cases}$$

3.3.1 Consumers' behaviour with buzz network effects

A consumer indexed by his valuation for the Information Good (θ) will buy the legitimate product under the condition that the utility he/she will receive from buying the good is higher than the utility he/she would receive from copying it. We define $\tilde{\theta}$ as the consumer who is indifferent between consuming the good legally or illegally. As $\theta - p_I + \beta x = \alpha\theta - (1-\alpha)M + \beta x$, we can find the cutoff:

$$\tilde{\theta} = \frac{p_I}{1-\alpha} - M \tag{3.1}$$

A consumer indexed by their valuation for the Information Good (θ) will illegally consume the product under the condition that the utility he/she receives from consuming the good is higher than the utility he would derive from not consuming the good at all. We define $\hat{\theta}$ as the consumer indifferent between consuming the good illegally or not consuming the good at all. As $\alpha\theta - (1-\alpha)M + \beta x = 0$ then we can find the cutoff:

$$\hat{\theta} = \frac{M(1-\alpha) - x\beta}{\alpha} = M(\frac{1}{\alpha} - 1) - \frac{1}{\alpha}x\beta \tag{3.2}$$

Note that in our model, buzz network effects make illegal downloading more attractive to consumers, as the cutoff $\hat{\theta}$ will shift toward the left with respect to a model without network effects.

This result is immediate and easy to spot when making a direct comparison with (2.14). We can thus deduce that buzz network effects make illegal downloading more attractive as they shift the cutoff $\hat{\theta}$ leftward.

Moving forward in our analysis, from those cutoff, we define consumers' type θ_0 if the consumers have $\theta \in [0, \hat{\theta}]$, we define consumers' type θ_p if the consumers have $\theta \in [\hat{\theta}, \tilde{\theta}]$ and we define consumers' type θ_b if the consumers have $\theta \in [\tilde{\theta}, 1]$.

When indexing consumers for their valuation for the Alternative Good, we face a dichotomous possibility: consumers type $\lambda_0 = \lambda$, for $\lambda \in [0, 1]$ will never be interested in consuming only the Information Good, while consumers type λ_b are willing to consume a bundle of the Information Good and of the Alternative Good.

As in this model we consider double heterogeneity, we will index our consumers by both their type $\{\theta, \lambda\}$. Thus we can say that in the indirect appropriation model with network effect there are five relevant consumers types, based on their combined valuation $\{\theta, \lambda\}$[3]:

[3]There should be six types of consumers, but consumers type $\{\theta_0, \lambda_b\}$ do behave as consumers type $\{\theta_0, \lambda_0\}$ due to our model assumption. For a in depth explanation, cross-reference Footnote

- Consumers type $\{\theta_0, \lambda_0\}$ will not consume either the Information Good or the Alternative Good.
- Consumers type $\{\theta_p, \lambda_0\}$ will prefer to consume the Information Goods illegally in the primary market and they are not interested in consuming the Alternative Goods.
- Consumers type $\{\theta_b, \lambda_0\}$ are willing to legally consume the Information Good in the primary market but they are not interested in consuming the Alternative Good.
- Consumers type $\{\theta_p, \lambda_b\}$ will prefer to consume the Information Goods illegally in the primary market, but they are interested in consuming the Alternative Goods.
- Consumers type $\{\theta_b, \lambda_b\}$ are willing to legally consume the Information Good in the primary market and they are interested in consuming the Alternative Good.

Notice that for piracy to be allowed in the market, we need to verify the condition $\tilde{\theta} > \hat{\theta}$. We will analyze this existence condition further in Subsection 4.5.

3.3.2 Copyright Owner's behaviour with buzz network effects

The copyright owner's problem in the indirect appropriation model with buzz network effects is to set the price for the Information Good (p_I), the royalties for the Alternative Good (p_A) and the piracy level in the market (α) such that he/she can maximize his/her profits (π), given the costs to produce the Information Good (C_I), the costs to sign the royalty agreement (C_A) and the cost to enforce digital piracy in the market (C_α). In a similar way to the previous model, we assume both the

12.

costs to produce the Information Good and the costs to sign the royalty agreement as fixed costs, and without loss of generality for simplicity's sake we will set them both equal to zero ($C_I = C_A = 0$), and we assume C_α to be a linear cost related to the level of digital piracy in the market. For simplicity's sake we will set it equals to the level of digital piracy in the market ($C_\alpha = \alpha$)[4].

Given this setting, we can write the monopolist problem as:

$$\max_{p_I, p_A, \alpha} \pi = D_I p_I + D_A p_A - C_I - C_A - C_\alpha \tag{3.3}$$

Where D_I is the demand for good I and D_A is the demand for good A.

3.3.3 Timing

Timing is unchanged with respect to the previous model.

In the first period, the copyright owner will set the level of digital piracy in the primary market, such that his profits will be maximized. In the second period the copyright owner will set the monopolistic prices for both the Information Good and the Alternative Good. In the third period, consumers will make their consumption choice in both markets based on their preference and on the buzz network effect.

3.4 Equilibrium Analysis with buzz network effects

In this Section we use backward induction to solve the digital piracy model with buzz network effects. In Subsection 4.1 we will introduce the evaluation of the expected consume volume x, in Subsection 4.2 we will analyze consumer's consumption choice given the expected consumption, in Subsection 4.3 we will find the monopolist's optimal pricing strategy and in Subsection 4.4 we will analyze the equilibrium piracy level in the market with buzz network effects. In Subsection 4.5 we will evaluate the existence conditions for piracy to be allowed in the model.

[4]for a detailed explanation on this settings, cross-reference Chapter 2.4.3

3.4.1 Evaluation of the buzz network effects

As noted above the buzz network effects affect all consumers who consume the good either legally or illegally.

From this concept, we define the expected level of consumers in the digital market as: $x \equiv 1 - \hat{\theta}$, where $\hat{\theta}$ is the consumer indifferent between consuming the good illegally or not consuming the Information Good at all. As we know $\hat{\theta}$ from (43), we can substitute into our definition of x as follows:

$$x = 1 - \frac{M(1 - \alpha -) - x\beta)}{\alpha}$$

Inverting for x, we obtain that the expected consumes for the digital market are:

$$x = \frac{\alpha - M(1 - \alpha)}{\alpha - \beta} \tag{3.4}$$

Notice that this implies that $\alpha \neq \beta$.

Moving forward, we can substitute the value (3.4) in (3.2). This allows us to evaluate the cutoff $\hat{\theta}$ with buzz network effects:

$$\hat{\theta} = \frac{M(1 - \alpha) - \beta}{\alpha - \beta} \tag{3.5}$$

In this model, the existence condition changes in $\tilde{\theta} > \hat{\theta}$ for $\alpha \neq \beta$, we will further analyze these existence conditions in Subsection 4.5.

3.4.2 Consumers' consumption choice with buzz network effects

Solving per backward induction, in time 2 consumers' will make their consumption choice. Thus, taking into account for the consumers' strategies $\{\theta, \lambda\}$, we can derive

the demand functions for the Information Good and the secondary good as follows:

$$D_{\{\theta,\lambda\}} = \begin{cases} D_{\{b,0\}} \equiv \lambda(1-\tilde{\theta}) & \text{generated by consumers type } \{\theta_b, \lambda_0\}, \text{ for the Information Good.} \\ D_{\{p,b\}} \equiv (1-\lambda)(\tilde{\theta}-\hat{\theta}) & \text{generated by consumers type } \{\theta_p, \lambda_b\}, \text{ for the Alternative Good.} \\ D_{\{b,b\}} \equiv (1-\lambda)(1-\tilde{\theta}) & \text{generated by consumers type } \{\theta_b, \lambda_b\}, \text{ for both goods.} \end{cases}$$

Thus, the monopolist will face, for good I, demand:

$$D_I = (1-\tilde{\theta}) \tag{3.6}$$

While he will face, for good A, demand:

$$D_A = (1-\lambda)(1-\tilde{\theta}) + (1-\lambda)(\tilde{\theta}-\hat{\theta})$$

Simplifying, we have that:

$$D_A = (1-\lambda)(1-\hat{\theta}) \tag{3.7}$$

To summarise, in time 2, in the model of indirect appropriation across complementary Information Goods with network effects, the consumers' consumption choice generates:

- For good I, demand: $D_I = (1-\tilde{\theta})$.
- For good A, demand: $D_A = (1-\lambda)(1-\hat{\theta})$

3.4.3 Copyright Owner's optimal pricing strategy with buzz network effects

Lemma 4 *Solving per backward induction, in time 1, in the indirect appropriation across complementary goods model with network effects, the Copyright Owners' optimal pricing strategies given the consumers' consumption choice stated previously are:*

- *the equilibrium price for the Alternative Good s.t. the Copyright Owners profits are maximized is:* $p_A^* = \psi$.

- *the equilibrium price for the Information Good s.t. the Copyright Owners profits are maximized is:* $p_I^* = \frac{1}{2}(1+M)(1-\alpha)$.

Thus, the equilibrium profits given the equilibrium price for the Information Good and the Alternative Good are: $\pi^* = \frac{1}{4}(1+M)^2(1-\alpha) - \alpha + \frac{\psi(1-\lambda)(\alpha - M(1-\alpha))}{\alpha-\beta}$

It is important to notice that the price in the primary market at time 1 in the indirect appropriation model and in the indirect appropriation with buzz network effects doesn't change, as buzz network effects are a clockwise pivot of the demand function.

From Subsection 3.2, we recall the copyright owner's monopolist problem as stated in Equation (3.3):

$$\max_{p_I, p_A, \alpha} \pi = D_I p_I + D_A p_A - C_I - C_A - C_\alpha$$

From Subsection 3.2 we recall $C_I = 0$ and $C_A = 0$ for simplicity's sake and that $C_\alpha = \alpha$, while from Subsection 3.4.2 we know D_I and D_A. We can thus rewrite the monopolistic problem as follow:

$$\max_{p_I, p_A} \pi = (1-\tilde{\theta})p_I + (1-\lambda)(1-\hat{\theta})p_A - \alpha \tag{3.8}$$

As the valuation of the secondary good ψ is common knowledge for the Copyright Owner, they will be able to set the maximum price when $V = 0$. As we know $V = \psi - p_A$, the Copyright Owner maximizes profits when $\psi - p_A = 0$. Thus we can derive that:

$$p_A^* = \psi \tag{3.9}$$

From 3.3.1 we can now recall 3.1 and 3.2:

$$\tilde{\theta} = \frac{p_I}{1-\alpha} - M \qquad\qquad \hat{\theta} = \frac{M(1-\alpha)-\beta}{\alpha-\beta}$$

Given these new Equations, we can rewrite Equation (3.8) as follow:

$$\max_{p_I} \pi = p_I(1 + M - \frac{p_I}{1-\alpha}) - \alpha + \frac{\psi(1-\lambda)(\alpha - M(1-\alpha))}{\alpha-\beta} \tag{3.10}$$

The first-order condition for profit maximization requires $\frac{\partial \pi}{\partial p_I} = 0$, we take the first derivative of the profit function, obtaining:

$$\frac{\partial \pi}{\partial p_I} = 1 + M - \frac{2p_I}{1-\alpha} \tag{3.11}$$

Setting (3.10) equal to zero and then solving for p_I, we find the equilibrium monopolistic price for the Information Good:

$$p_I^* = \frac{1}{2}(1+M)(1-\alpha) \tag{3.12}$$

Notice that the equilibrium price of the Information Good is actually the same in the model with buzz network effects and in the model without network effects. This happens as the demand with network effects is a clockwise pivot of a linear demand function, thus leading to the same monopoly price. Given those results, we can rewrite the profit function given the equilibrium prices as:

$$\pi^* = \frac{1}{4}(1+M)^2(1-\alpha) - \alpha + \frac{\psi(1-\lambda)(\alpha - M(1-\alpha))}{\alpha-\beta} \tag{3.13}$$

3.4.4 Copyright Owner's optimal digital piracy level in the primary market

Proposition 2 *Solving for backward induction, in the indirect appropriation across complementary goods model with network effects, at time 0 there exists a positive level of digital piracy in the market s.t. the monopolist profits are maximized, such level being:* $\alpha^* = \beta + \frac{2\sqrt{-\psi(1-\lambda)(5+M(2+M))(\beta-M(1-\beta))}}{5+M(2+M)}$

Similarly to what shown in Chapter 2, as long as digital piracy exists in the market, α^* is an internal solution of the model. Thus, as long as digital piracy is allowed in the market it also exists an equilibrium level of digital piracy s.t. the copyright owner's monopolist profits are maximized.

Lemma 5 *Given the equilibrium level of digital piracy in the market shown in the previous proposition, in time 0:*

- *the equilibrium price for the Information Good s.t. the Copyright Owners profits are maximized is:* $p_I^{**} = \frac{(1+M)(5+M(2+M)(1-\beta)-5\beta-2\sqrt{\psi(5+M(2+M))(M(1-\beta)-\beta)(1-\lambda)})}{2(5+M(2+M))}$
- *the equilibrium profits given the equilibrium price for the Information Good and the Alternative Good are:*

$$\pi^{**} = \frac{1}{4}(1 + M(2+M)(1-\beta) - 5\beta + 4\psi + 4\psi(M(1-\lambda) - \lambda)) \\ - 4\sqrt{\psi(5 + M(2+M))(M(1-\beta) - \beta)(1-\lambda)}$$

This result holds under the following conditions: $\alpha^* \in [0,1]$ for all $\beta \in [0, \frac{1}{2}]$, $\lambda \in [0,1]$, $\psi > 0$ and $M > \frac{\beta}{1-\beta}$.

From Equation 3.9, 3.10 and 3.12 we can rewrite the copyright owner's monopolist profit as:

$$\max_{\alpha} \pi = \frac{1}{4}(1+M)^2(1-\alpha) - \alpha + \frac{\psi(1-\lambda)(\alpha - M(1-\alpha))}{\alpha - \beta} \tag{3.14}$$

As the first order condition for this maximization problem is $\frac{\partial \pi}{\partial \alpha} = 0$, we thus obtain:

$$\frac{\partial \pi}{\partial \alpha} = \frac{\psi(1+M)(1-\lambda)}{\alpha-\beta} - 1 - \frac{1}{4}(1+M)^2 - \frac{\psi(1-\lambda)(\alpha - M(1-\alpha))}{(\alpha-\beta)^2} \tag{3.15}$$

Setting this Equation equal to zero and solving for the piracy level α, we obtain two possible solutions:

$$\alpha = \beta + \frac{2\sqrt{-\psi(1-\lambda)(5+M(2+M))(\beta - M(1-\beta))}}{5+M(2+M)}$$
$$\alpha = \beta - \frac{2\sqrt{-\psi(1-\lambda)(5+M(2+M))(\beta - M(1-\beta))}}{5+M(2+M)}$$

Notice that $-\psi(1-\lambda)(5+M(2+M))(\beta - M(1-\beta)) > 0$ for all $\beta \in [0,1]$, $\lambda \in [0,1]$, $\psi > 0$ and $M > \frac{\beta}{1-\beta}$.

It is also important to underline that although both solutions may yield $\alpha \in [0,1]$ for $\beta \neq 0$, only the first function admits a solution also for $\beta = 0$. Also, ceteris paribus, the first solution will in all cases lead to a higher level of digital piracy in the primary market. Following a principle of prudence, we will thus consider the "worst scenario" where digital piracy is higher in the market. The equilibrium level of digital piracy in the buzz network effects model is:

$$\alpha^* = \beta + \frac{2\sqrt{-\psi(1-\lambda)(5+M(2+M))(\beta - M(1-\beta))}}{5+M(2+M)} \tag{3.16}$$

Notice that $\alpha^* \in [0,1]$ for all $\beta \in [0, \frac{1}{2}]$, $\lambda \in [0,1]$, $\psi > 0$ and $M > \frac{\beta}{1-\beta}$. From this condition we know that there exists an equilibrium level of digital piracy in the digital market only for sufficiently weak buzz network effects.

Given the equilibrium level of piracy in the primary market and the equilibrium price for the Information Good, we can derive:

$$p_I^{**} = \frac{(1+M)(5+M(2+M)(1-\beta) - 5\beta - 2\sqrt{\psi(5+M(2+M))(M(1-\beta)-\beta)(1-\lambda))}}{2(5+M(2+M))} \tag{3.17}$$

Substituting (3.16) and (3.17) into the monopolist profit function, we obtain the equilibrium copyright owner's profits:

$$\pi^{**} = \frac{1}{4}(1 + M(2+M)(1-\beta) - 5\beta + 4\psi + 4\psi(M(1-\lambda) - \lambda)) \\ - 4\sqrt{\psi(5 + M(2+M))(M(1-\beta) - \beta)(1-\lambda)} \quad (3.18)$$

3.4.5 Existence of Piracy with buzz network effects

While solving the model, we derived the following condition: $\alpha \neq \beta$, $\beta \in [0, \frac{1}{2}]$ and $M > \frac{\beta}{1-\beta}$. Also, Notice that as in the previous model we also need to satisfy the condition $\tilde{\theta} > \hat{\theta}$ for piracy to be allowed in the market.
From (3.1) and (3.5), we can rewrite the condition $\tilde{\theta} > \hat{\theta}$ as:

$$\frac{p_I}{1-\alpha} - M > \frac{M(1-\alpha-\beta)}{\alpha-\beta}$$

Also, as we know the equilibrium level of digital piracy and the equilibrium price in the primary market, we can derive:

$$\frac{1-M}{2} > -\frac{\sqrt{\psi(1-\lambda)(5 + M(2+M))(\beta - M(1-\beta))}}{2\psi(1-\lambda)} \quad (3.19)$$

Under the model conditions: $\alpha \neq \beta$, $\beta \in [0, \frac{1}{2}]$ and $M > \frac{\beta}{1-\beta}$, $\psi \in [0,1]$ and $\lambda \in [0,1]$, in this model exists an interior solution for an optimal level of digital piracy that allows for copyright owner's profits:

- for fine $M \in (0, \bar{M})$ and consumers' valuation for the Alternative Good $\psi \in (\underline{\psi}, 1)$ and fraction of consumers interested only in consumption of the Information Goods $\lambda \in (0, \bar{\lambda})$ and network effects intensity $\beta \in (0, \bar{\beta})$

- for fine $M \in (0, \bar{M})$ and consumers' valuation for the Alternative Good $\psi \in (0, \underline{\psi}]$ and fraction of consumers interested only in consumption of the

Information Goods$\lambda \in (0,1)$ and network effects intensity $\beta \in (\tilde{\beta}, \bar{\beta})$

- for fine $M \in (0, \bar{M})$ and consumers' valuation for the Alternative Good $\psi \in (\underline{\psi}, 1)$ and fraction of consumers interested only in consumption of the Information Goods$\lambda \in (\bar{\lambda}, 1)$ and network effects intensity $\beta \in (\tilde{\beta}, \bar{\beta})$

- for fine $M \in (\bar{M}, 1)$ and consumers' valuation for the Alternative Good $\psi \in (0,1)$ and fraction of consumers interested only in consumption of the Information Goods$\lambda \in (0,1)$ and network effects intensity $\beta \in (\tilde{\beta}, \bar{\beta})$

Where: $\bar{M} = 0.29559774252208476$, $\bar{\psi} = \frac{M(5+M(2+M))}{(1+M)^2}$, $\bar{\lambda} = 1 - \frac{M(5+M(2+M))}{\psi(1+M)^2}$,$\tilde{\beta} = \frac{M}{1+M} - \frac{\psi(1-\lambda)(M+1)}{5+M(2+M)}$, $\bar{\beta} = \frac{M}{1+M}$.

For simplicity's sake, in the following analysis, for network effects intensity $\beta \in (0, \bar{\beta})$ we will simply state that network effects exist in the model, for network effects intensity $\beta \in (\tilde{\beta}, \bar{\beta})$ we will state that strong network effect intensity is required in the market, for $\lambda \in (0, \bar{\lambda})$ we will instead refer to a sufficiently low fraction of consumers interested only in consumption of the Information Goods, for $\lambda \in (\bar{\psi}, 1)$ we will instead refer to a sufficiently high fraction of consumers interested only in consumption of the Information Goods, for $\lambda \in (0,1)$ we will instead refer to a any fraction of consumers interested only in consumption of the Information Goods, for $\psi \in (\bar{\psi}, 1)$ we will refer to a sufficiently high consumers' valuation for the Alternative Good, for $\psi \in (0, \bar{\psi}]$ we will refer for sufficiently low consumers' valuation for the Alternative Good and for $\psi \in (0,1]$ we will refer for sufficiently low consumers' valuation for the Alternative Good.

Before moving forward in the analysis, it is important to Notice that we will refer to the conditions in order of exposition in the previous bulleted list (from the first in exposition to last, will be referred to as condition 1 to 4). Also, it is important to Notice that condition 1 values ($\bar{M}$, $\psi \in (\underline{\psi}, 1)$ and $\lambda \in (0, \bar{\lambda})$) fully correspond to the ones shown in Chapter 2.

The first, important, insight we obtain from this existence condition is that the

introduction of buzz network effects in the market allows for laxier existancy conditions for the equilibrium. In detail, condition 1 for existence states that whenever the conditions as per Chapter 2 are present in the market, and as long as network effects are present in the market, then an internal solution for equilibrium is possible. This condition is of paramount importance in our work, as it clearly shows that indirect appropriation across complementary products is indeed compatible with the presence of network effects in the market, as at the best of our knowledge this is the first theoretical model which includes both effects.

Condition 2 further the reach of possible equilibrium for the model, as it shows that as long as there are strong network effects and the fine is sufficiently low, then even if there is low consumers' valuation for the Alternative Good and independently from the fraction of consumers interested only in consuming the Information Good an equilibrium can be reached. This condition is then extended in Condition 3, stating that when consumers' valuation for the Alternative Good is high, then even if there is a high fraction of consumers interested only in consuming the Primary Good in the market an equilibrium can be reached.

Although we argued the paramount importance of condition 1 for our model, condition 4 may well share the same importance. This existancy condition implies that as long as there are strong enough network effects in the market, and as long as the fine is high, then there will always be an equilibrium as long as there is an alternative market. The full relevance of this condition consists in its policy implications, as it confirms and reveal the insight we already had in the previous model: the fine (M) is an inadequate policy instrument to reduce digital piracy. In the previous model, we showed that as long as digital piracy is allowed in the market, then it is preferable to have low fine (M) as an increase in fine leads to lower copyright owner's profits. In this Chapter not only we demonstrate that, but also we saw that when strong network effects are present in the market, if the fine overcome the threshold $\bar{M}$ instead of eliminating digital piracy from the market similarly to

what we saw in the previous model, we have the opposite effect: there will always be an internal equilibrium in the model or - in simpler terms - it will always exists a level of digital piracy that allow for copyright owner's profits maximization and there will always be digital piracy in the market.

In Chapter two we wondered why although digital piracy could theoretically be excluded from the market through the fine (M) in the real world the enforcement efforts of copyright owners and law enforcement agencies brought to less than satisfactory results[5]. This result offers us an insight on a mechanism which could offer an alternative and new explaination on the problem: due to the presence of strong network externalities in the market, the fine M is completely inadequate and cannot reduce digital piracy.

We can summarise these results as follow.

When in a market with indirect appropriation across complementary Information Goods are present network effects, then the conditions which allows for an internal solution in the model are the following conditions.

Condition 1: for fine $M \in (0, \bar{M})$ and consumers' valuation for the Alternative Good $\psi \in (\underline{\psi}, 1)$ and fraction of consumers interested only in consumption of the Information Goods $\lambda \in (0, \bar{\lambda})$ and network effects intensity $\beta \in (0, \bar{\beta})$

Condition 2: for fine $M \in (0, \bar{M})$ and consumers' valuation for the Alternative Good $\psi \in (0, \underline{\psi}]$ and fraction of consumers interested only in consumption of the Information Goods$\lambda \in (0, 1)$ and network effects intensity $\beta \in (\tilde{\beta}, \bar{\beta})$

Condition 3: for fine $M \in (0, \bar{M})$ and consumers' valuation for the Alternative Good $\psi \in (\underline{\psi}, 1)$ and fraction of consumers interested only in consumption of the Information Goods$\lambda \in (\bar{\lambda}, 1)$ and network effects intensity $\beta \in (\tilde{\beta}, \bar{\beta})$

Condition 4: for fine $M \in (\bar{M}, 1)$ and consumers' valuation for the Alternative Good $\psi \in (0, 1)$ and fraction of consumers interested only in consumption of the Information Goods$\lambda \in (0, 1)$ and network effects intensity $\beta \in (\tilde{\beta}, \bar{\beta})$

[5]as argued in Chapter 2 in the introduction, in the existancy conditions for digital piracy sections and in the comparative statics sections and then discussed in Section 2.7.

Where: $\bar{M} = 0.29559774252208476$, $\bar{\psi} = \frac{M(5+M(2+M))}{(1+M)^2}$, $\bar{\lambda} = 1 - \frac{M(5+M(2+M))}{\psi(1+M)^2}$,$\tilde{\beta} = \frac{M}{1+M} - \frac{\psi(1-\lambda)(M+1)}{5+M(2+M)}$, $\bar{\beta} = \frac{M}{1+M}$.

From this, we can state that when network effects are present in an environment where indirect appropriation across complementary product market is possible, then the conditions to have a intern solution for a level of digital piracy which maximizes the copyright owner's profits are laxier than in a model where we have no network externalities.

Note that under condition 1, when the set of conditions for piracy existency in the model without network effects are realized, then there will be a internal solutions as long as network effects are added in the market.

Also, given condition 2 and 3, when strong network effects are present in the market we can have an internal solution for smaller alternative markets w.r.t. the ones we analyzed in Chapter 2.

Given condition 4, when strong network effects are present in the market and the fine is higher than the threshold $\bar{M}$, then the model have always an interior solution independently as long as there is an alternative market.

Last but not least, from condition 4 we can state that the fine M is an inadequate way to reduce digital piracy in a market in which is possible to have indirect appropriability across complementary products and network effects.

3.5 Comparative statics with Buzz Network Effects

In this Section we analyze the statics of the main results from this model in order to focus on the interactions between variables in the equilibrium. In Subsection 5.1 we will address the comparative statics on the level of digital piracy that maximizes the copyright owner's profits (α^*), in Subsection 5.2 we will address the comparative statics on the equilibrium price for the Information Good given the equilibrium level of digital piracy in the digital market (p_I^{**}) and in Section 5.3 we will study the

comparative statics on the equilibrium profits for the Copyright Owner. We will not address the comparative statics on the equilibrium price for the Information Good in time one as it has already been addressed in Chapter 2[6].

3.5.1 Comparative statics on the equilibrium level of digital piracy with network effects

The first step in our analysis will be evaluating the comparative statics on the equilibrium level of piracy in the primary market α^*. We recall Equation (3.16):

$$\alpha^* = \beta + \frac{2\sqrt{-\psi(1-\lambda)(5+M(2+M))(\beta - M(1-\beta))}}{5+M(2+M)}$$

The variables that affect the equilibrium level of α are the intensity of network effect β, the fine M, the consumers' valuation for the Alternative Good ψ and the level of interest in buying the bundle of the intellectual and Alternative Goods $\lambda_b = 1 - \lambda$. Also, Notice that to allow for piracy to exist in the market, we have to respect the conditions shown in Subsection 4.5.

The first effect we will analyze is the impact of the cultural buzz network effects intensity β on the equilibrium level of digital piracy.

$$\frac{\partial \alpha^*}{\partial \beta} = 1 - \frac{\psi(1+M)(1-\lambda)}{\sqrt{\psi(5+M(2+M))(M(1-\beta)-\beta)(1-\lambda)}} \tag{3.20}$$

Under the four existancy conditions as per Subsection 4.5, this function is always negative. Thus, a marginal increase in the buzz network effects will lead to a decrease in the level of digital piracy in the market.

Moving forward in our analysis, we will now evaluate the effect of the fine (M) on

[6]In both models the equilibrium price at time 1 is the same in both models

the level of digital piracy.

$$\frac{\partial \alpha^*}{\partial M} = \frac{\psi(1-\lambda)(5-M^2+\beta(M-1)(M+3))}{(5+M(2+M))\sqrt{\psi(1-\lambda)(5+M(2+M))(M(1-\beta)-\beta)}} \tag{3.21}$$

Under the four existancy conditions as per Subsection 4.5, this function is always positive. Thus *ceteris paribus* a marginal increase in the fine M will lead to an increase in the equilibrium level of digital piracy in the primary market. This result is analogous to the one seen in the previous Chapter.

We will now evaluate the comparative statics for the fraction of consumers interested only in consuming the Information Good (λ).

$$\frac{\partial \alpha^*}{\partial \lambda} = -\frac{\psi(M(1-\beta)-\beta)}{\sqrt{\psi(1-\lambda)(5+M(2+M))(M(1-\beta)-\beta)}} \tag{3.22}$$

Under the four existancy conditions as per Subsection 4.5, this function is always positive. Thus a marginal increase in the fraction of consumers interested only in consuming the Information Good λ will lead to a decrease in the equilibrium level of the piracy. This result is coherent with the result shown in the previous Chapter.

The last variable we study is the consumers' valuation for the Alternative Good (ψ).

$$\frac{\partial \alpha^*}{\partial \psi} = \frac{(1-\lambda)(M(1-\beta)\beta)}{\sqrt{\psi(1-\lambda)(5+M(2+M))(M(1-\beta)-\beta)}} \tag{3.23}$$

Under the four existancy conditions as per Subsection 4.5, this function is always positive. Thus, a marginal increase in consumers' valuation for the Alternative Goods will lead to an increase in the equilibrium level of the piracy. This result is coherent with the results shown in the Chapter 2.

While it is still early to discuss the implication of the results of the impact of buzz network effect intensity on the level of digital piracy in the market, this result may appear puzzling at first in a similar way to what happened to the fine in Chapter 2. As we know that the cutoff $\hat{\theta}$ will move leftward in presence of buzz network effect

with respect to the model without network effect, we should expect digital piracy to increase in the market as the higher the intensity of network effect is. This insight is strenghten if we account for the principle that buzz network effect allows for laxier existence conditions for digital piracy existence in the model. In stark contrast with the previous insights, this negative impact of buzz network effect intensity on digital piracy appears counterintuitive. We will further develop on this topic in Section 7. Note that the other results are instead coherent with what shown in Chapter 2.

3.5.2 Comparative statics on equilibrium price in the primary market with network effects

We will now proceed to evaluate the effect of our variables of the equilibrium price in the primary market given the equilibrium level of piracy in the market p_I^{**}. We recall from (3.18):

$$p_I^{**} = \frac{(1+M)(5+M(2+M)(1-\beta)-5\beta-2\sqrt{\psi(5+M(2+M))(M(1-\beta)-\beta)(1-\lambda)})}{2(5+M(2+M))}$$

The four variables that have effect on the price p_I^{**} are the intensity of buzz network effects β, the fine M, the consumers' valuation for the Alternative Good ψ and the level of interest in the Alternative Good expressed through λ.

We will now analyze the effect of a marginal increase in the intensity of buzz network effects on the equilibrium price:

$$\frac{\partial p_I^{**}}{\partial \beta} = \frac{(1+M)(\psi(1+M)(1-\lambda)-\sqrt{\psi(1-\lambda)(5+M(2+M))(M(1-\beta)-\beta)})}{2\sqrt{\psi(1-\lambda)(5+M(2+M))(M(1-\beta)-\beta)}} \tag{3.24}$$

Under the four existancy conditions as per Subsection 4.5, this function is always positive. This implies that a marginal increase in the intensity of the buzz network effects (β) will lead to an increase in equilibrium price in the primary market[7].

[7] see appendix for the constraint of this function.

Moving forward in our analysis, we evaluate the implications of the fine M on the equilibrium price.

$$\frac{\partial p_I^{**}}{\partial M} = \frac{1}{2}(1 - \beta + \frac{\psi(1-\lambda)(13\beta - 5 - M(15 + M(3+M))(1-\beta))}{(5 + M(2+M))\sqrt{\psi(1-\lambda)(5+M(2+M))(M(1-\beta)-\beta)}}) \tag{3.25}$$

Under the four existancy conditions as per Subsection 4.5, this function is always negative. This implies that a marginal increase in the fine (M) will lead to a decrease in the equilibrium price. This result is in line with what we expect from Chapter 2 on the mechanics between fine and equilibrium[8].

The third variable we analyze is the fraction of consumers interested only in consuming the Information Good (λ)

$$\frac{\partial p_I^{**}}{\partial \lambda} = \frac{\psi(1+M)(M(1-\beta)-\beta)}{2\sqrt{\psi(1-\lambda)(5+M(2+M))(M(1-\beta)-\beta)}} \tag{3.26}$$

Under the four existancy conditions as per Subsection 4.5, this function is always positive. This implies that a marginal increase in the fraction of consumers interested only in consuming the Information Good will lead to an increase in the equilibrium level price for the Alternative Good. This result is in line with what we expect for what is shown in Chapter 2.

The last variable with impact on the equilibrium price is the consumers' valuation for the Alternative Good (ψ)

$$\frac{\partial p_I^{**}}{\partial \psi} = -\frac{(1-\lambda)(1+M)(M(1-\beta)-\beta)}{2\sqrt{\psi(1-\lambda)(5+M(2+M))(M(1-\beta)-\beta)}} \tag{3.27}$$

Under the four existancy conditions as per Subsection 4.5, this function is always negative. This implies that a marginal increase in the consumers' valuation for the Alternative Good will lead to a decrease in the equilibrium price. This result is in line with what we expect from Chapter 2 on both the benchmark model and the

[8] see appendix for the constraint of this function.

indirect appropriation model[9].

This finding gives some indication of the strength of buzz network effect on a market with indirect appropriation across complementary products: the ability to *ceteris paribus* rise the equilibrium price in the primary market is important and shows that in the real world a correct estimation of this variable may have important implications for copyright owners. On the other hand, the results of the other variables are in line with what already discussed in Chapter 2.

3.5.3 Comparative statics on the equilibrium copyright owner's profits with network effects

The last equilibrium we analyze in this extension model is the copyright owner's monopolistic profits in equilibrium. To do so, we recall the function from (3.18): $\pi^* = \frac{1}{4}(1+M)^2(1-\alpha) - \alpha + \frac{\psi(1-\lambda)(\alpha - M(1-\alpha))}{\alpha - \beta}$ It is possible to see that four variables affect the copyright owner's monopolistic profits in equilibrium: the intensity of buzz network effects β, the fine M, the consumers' valuation for the Alternative Good ψ and the level of interest in the Alternative Good expressed through λ.

As in the previous Section, we will start studying the effect of the intensity of buzz network effects on the profits.

$$\pi^{**} = \frac{1}{4}(1 + M(2+M)(1-\beta) - 5\beta + 4\psi + 4\psi(M(1-\lambda) - \lambda)) \\ - 4\sqrt{\psi(5 + M(2+M))(M(1-\beta) - \beta)(1-\lambda)} \quad (3.28)$$

Under the four existancy conditions as per Subsection 4.5, this function is always positive. This means that a marginal increase in the intensity of buzz network effects (β) will result in an increase in profits for the copyright owner. Moving forward in our analysis, we will now consider the effect of the fine (M) on the copyright owner's

[9] see appendix for the constraint of this function.

equilibrium profits.

$$\frac{\partial \pi^{**}}{\partial M} = \frac{\psi(1-\lambda)(7\beta - 5 + M(3M(\beta-1)+6\beta-4) + 2\sqrt{\psi(5+M(2+M))(M(1-\beta)-\beta)(1-\lambda)})}{2\sqrt{\psi(5+M(2+M))(M(1-\beta)-\beta)(1-\lambda)}} + \frac{1}{2}(1+M)(1-\beta) \quad (3.29)$$

Under the four existancy conditions as per Subsection 4.5, this function is always negative. Thus an increase in the fine M will lead to a decrease in the equilibrium copyright owner's profits. This result conforms with the insight gained in Chapter 2 on the benchmark model and on the indirect appropriation model.

We will now evaluate the effect of a marginal increase in the fraction of consumers interested in consuming only the Information Good (λ) on the equilibrium level of profits.

$$\frac{\partial p_I^{**}}{\partial \lambda} = \frac{\sqrt{\psi(5+M(2+M))(M(1-\beta)-\beta)(1-\lambda)}}{2(1-\lambda)} - \psi(1+M) \quad (3.30)$$

Under the four existancy conditions as per Subsection 4.5, this function is always negative. Thus, a marginal increase in the fraction of consumers interested only in the Information Good (λ) yields a decrease in the equilibrium level of copyright owner's profits. This result is expected in the light of what shown in the previous Chapter.

Moving forward, we will now study the effect of a marginal increase in consumers' valuation for the alternative market (ψ) on the equilibrium level of profits.

$$\frac{\partial \pi^{**}}{\partial \psi} = (1+M)(1-\lambda) - \frac{\sqrt{\psi(5+M(2+M))(M(1-\beta)-\beta)(1-\lambda)}}{2\psi} \quad (3.31)$$

Under the four existancy conditions as per Subsection 4.5, this function is always positive. Thus, a marginal increase in the consumers' valuation for the Alternative Good (ψ) yields an increase in equilibrium level of the copyright owner's profits. Once again, this result falls in line with the insights shown in Chapter 2.

Before moving forward, we would like to share an observation derived from the analysis of the cross-effect of a marginal increase in both the consumers' valuation for the Alternative Good (ψ) and in the fraction of consumers interested only in consuming the Information Good (λ). Notice that by the Schwarz Theorem there is symmetry of the second derivatives, thus $\frac{\partial^2 \pi^{**}}{\partial \lambda \partial \psi} = \frac{\partial^2 \pi^{**}}{\partial \psi \partial \lambda}$.

$$\frac{\partial^2 \pi^{**}}{\partial \lambda \partial \psi} = -1 - M + \frac{\sqrt{\psi(5 + M(2 + M))(M(1 - \beta) - \beta)(1 - \lambda)}}{4\psi(1 - \lambda)} \tag{3.32}$$

Under the four existancy conditions as per Subsection 4.5, this function is always negative.

While from previous literature on network effects we could already expect that a marginal increase in network effects would lead to higher profits for the copyright owner, this result is nethertheless interesting and important. To the best of our knowledge and at the time of writing, theory on indirect appropriation across products has not been combined with literature on network effects from a theoretical standpoint. Our result thus confirms that the two effects are indeed compatible, and this is an achievement in and of itself. Last but not least, we have confirmation that all the effects we observed in the benchmark model and in the indirect appropriation model carry over when network effects are present in the market.

3.6 Comparison between the results in the indirect appropriation model with and without network effects

We will now make a brief comparison between the equilibrium values shown in the two models. As the model with network effects allows for a higher degree of existance, the limitations we will apply in this analysis will be the ones as per

Chapter 2.5.7 instead of the ones presented in this Chapter.

Under the restrictions as per Chapter 2.5.7 and as long as the intensity of network effects in the market is constrained as ($\beta \in (0, \bar{\beta})$), the equilibrium level of digital piracy in the primary market will always be higher in the model without network effects, while both the equilibrium price in the primary market and the copyright owner's profits in equilibrium will be lower in the model without network effects.

This result underline the positive influence of network effects in the market: ceteris paribus copyright owners will in fact face less digital piracy in the primary market, while offering the digital good at an higher equilibrium price while earning higher profits in equilibrium.

We extend this comparison to the effects we introduced in the benchmark model: the value effect, the interest effect and the virality effect. Under the restrictions as per Chapter 2.5.7 and as long as the intensity of network effects in the market follow the constraint ($\beta \in (0, \bar{\beta})$), the analysis yield that the both the value effect and the virality effect will be stronger in the market without network effect, while the interest effect will be weaker in the model without network effects.

This result is quite interesting: the introduction of network in the market weakens the influence of consumers' valuation on the copyright owner's profits, while at the same time it strenghten the influence of the fraction of consumers willing to consume only the Information Good on said profits. The virality effect is also weakened by the presence of network effects. We can deduce that while the introduction of network effect in the model has positive effects on the results, it still somewhat weakens the basic mechanics underlying complementarity between the primary and the Alternative Good.

When comparing a model of indirect appropriation across products with or without network effects, *ceteris paribus* and as long as we face an intensity of network effects $\beta \in (0, \bar{\beta})$ we:

- the equilibrium level of piracy in the primary market will be lower in the

model without network effects

- the equilibrium price in the primary market will be lower in the model without network effects
- the copyright owner's equilibrium profits will be lower in the model without network effects

Also, when evaluating the strenght of the effect we introduced in the benchmark model and their relationship with network effects, we find out that:

- the value effect is stronger in the model without network effects
- the interest effect is weaker in the model without network effects
- the virality effect is weaker in the model without network effects

From this results, we conclude that although the introduction of network effects in the market yields better results for the copyright owner, at the same time it weakens the complementarity between the Primary Good and the Alternative Good.

3.7 Result discussion

Now that we have analyzed all the main mechanisms behind this model, we can evaluate and discuss more closely the effects of the introduction of buzz network effects in the indirect appropriation model for complementary Information Goods. Since the initial steps in the model we saw a leftward shift of the cutoff $\hat{\theta}$ with respect to the previous model. As $\hat{\theta}$ represents the consumer indifferent between illegally consuming the digital goods and not consuming at all, we could expect that buzz network effects would have a strengthening effect toward digital piracy. The expectations were once again raised while analyzing the existence condition from the model. From our results, we can deduce how the presence of buzz network effects extends the robustness of the model well over what was allowed in the model

for indirect appropriation across complementary products. The limits of the existence condition implies that as long as there are strong enough buzz network effects and as long as there exists an alternative market, the model will always have an interior solution, thus granting that there would always be an equilibrium level of digital piracy which allows for copyright owner's profits maximization. This result leads to two main implications: on one hand, firms which operate in a market with strong network effects and with alternative market have more probability to find the optimal level of digital piracy which maximizes their profits. On the other hand, it implies that in a market with those characteristics it may well be impossible to push piracy out of the market, fact that is coherent with the observed data in the real world: notwithstanding the effort of law enforcement and firms, digital piracy is still growing. Our results also show that a higher intensity in network effects may lead, *ceteris paribus*, to lower level of digital piracy in the primary market, to higher prices in the primary market or to higher profits. This result is exactly the opposite of what shown in Chapter two for the fine (M): while a firm would prefer less enforcement in the form of the fine (M), it would warmly welcome higher intensity of network effects. From a mechanic point of view, we can deduce that the equilibrium level of digital piracy in the primary market and the equilibrium price in the primary market are connected by a inverse relationship and the firm can use one or the other to compensate in undesired changes.

The second most important insight is that the fine (M) might not be adequate as an instrument to limit digital piracy in a market where there are buzz network effects: when high enough buzz network effects are present in the market, then even for the maximum fine possible digital piracy would still be present in the market. In Chapter 2 we wondered about the possible reasons why digital piracy is so widespread even though it is theoretically possible to exclude it through law enforcement and, in our case, through fines. One of the hypotheses was that the value consumers gain from digital piracy is so high that the fine is not enough to scare them, and

this result seems to point exactly in that direction. This also implies strong implications in the real world, as it offers a possible explanation for the inefficiency of law enforcement intervention in deterring piracy worldwide. Notice that this type of result is coherent with what is being shown in economic literature[10]. Also, note that this result strengthen the conclusions we made in our previous model on the need of a new and more inclusive system of copyright enforcement: under the condition of this model, to allow for digital piracy to be an instrument in the hand of copyright owner, setting the best fine (M) and the best optimal level of piracy (α) is of paramount importance for copyright owners when facing digital piracy to allow for profit maximization.

As far as the interaction between indirect appropriation and network effects goes, we also see that the two effects can indeed coexist in the market, leading to a positive effect for the copyright owner. Once again, this result is confirmed by the existence of an internal equilibrium for the model, which implies that digital piracy may be the profit maximizing strategy for the firm.

Last but not least, our results offer quite a good insight on how indirect appropriation interact with network effects. When network effects are present in the market, the copyright owner will face less digital piracy in the primary market, higher price in the primary market and higher profits w.r.t. the model without network effects. Thus we can conclude that it is desiderable for copyright owners to have access to this positive externalities. This result is quite coherent with the constant efforts of firms to cultivate and enlarge their fandom over time. On the other hand, the model shows that network effects weakens the mechanism of complementarity between the Primary Good and the Alternative Good. While this does lead to better overall results, it also shows that there is a limit in compatibility between the two different way to exploit digital piracy. Once again, this result strengthens the opinion we

[10] one of the contributions on this topic being Peuker et al. (2017), in which the authors show from evidence registered after the closure of MegaUpload. They also conclude that a blanket copyright enforcement policy may be inappropriate if products are heterogeneous.

presented in Chapter 2 on the dire need of a more flexible enforcement system for IP rights, where authors and copyright owners could have a more decisive and relevant role on deciding the strength and the limit of copyright enforcement on a personal basis.

3.8 Limitations of the model, future developments and concluding remarks

The model presented several challenges during formulation, the main challenge being the discontinuity in the level of digital piracy in the primary market and in the intensity of network effects due to the constraints shown while evaluating $\hat{\theta}$ which greatly complicates the analysis of the existancy conditions and in the comparison between the model with and without network effects. Also, notice that although the insights brought by this model are significant, it remains relatively unrefined and needs to be perfected in the future. At first sight, this may appear to be a minor achievement, but as far as we know there is no existing theoretical model that allows for indirect appropriation across products and network effects to coexist and to interact. This researched used a simple but robust framework upon which we build a bridge into unknown territory, but this of course implies that there are wide margins for improvement. The first and most important direction of development would be the introduction of a social welfare analysis into the benchmark model, the indirect appropriation model and the indirect appropriation model with network effects. This could allow for stronger policy considerations.

In conclusion, this model succeeded in proving that in a digital market where digital piracy is present, the combined effect of indirect appropriation across complementary products and network effects allow for profit maximization. It also proves that the presence of network effects on the market lessen the pressure of digital piracy for the firms, thus allowing for laxier equilibrium existence requirements. Amongst

the results we found, an important insight is the inefficiency of the fine (M) as a deterrent against piracy.

Appendix

3.9 Value of the threshold $\dot{M}$

$$\dot{M} = \{-11 - 160\psi^2(1-\lambda)^2 - 2[37 - 2\psi(1-\lambda)(129 + 4\psi(1-\lambda)(39 - 251\psi(1-\lambda))) + 6\sqrt{75 - 3\psi(1-\lambda)(123 - \psi(1-\lambda)(-57 - 4\psi(1-\lambda)(-355 - 2\psi(1-\lambda)(-251 - 2\psi(1-\lambda)(14 + 37\psi(1-\lambda))))))}]^{\frac{1}{3}}$$
$$+ [37 - 2\psi(1-\lambda)(129 + 4\psi(1-\lambda)(39 - 251\psi(1-\lambda)) + 6\sqrt{75 - 3\psi(1-\lambda)(123 - \psi(1-\lambda)(-57 - 4\psi(1-\lambda)(-355 - 2\psi(1-\lambda)(-251 - 2\psi(1-\lambda)(14 + 37\psi(1-\lambda))))))}]^{\frac{2}{3}}$$
$$+ 16\psi(1-\lambda)(-1 + (37 - 2\psi(1-\lambda)(129 + 4\psi(1-\lambda)(39 - 251\psi(1-\lambda))) + 6\sqrt{75 - 3\psi(1-\lambda)(123 - \psi(1-\lambda)(-57 - 4\psi(1-\lambda)(-355 - 2\psi(1-\lambda)(-251 - 2\psi(1-\lambda)(14 + 37\psi(1-\lambda))))))}]^{\frac{1}{3}}))]$$
$$/[3(37 - 2\psi(1-\lambda)(129 + 4\psi(1-\lambda)(-39 + 251\psi(1-\lambda)))) + 6\sqrt{75 - 3\psi(1-\lambda)(123 - \psi(1-\lambda)(-57 - 4\psi(1-\lambda)(-355 - 2\psi(1-\lambda)(-251 - 2\psi(1-\lambda)(14 + 37\psi(1-\lambda))))))}]^{\frac{1}{3}}\}$$

Notice that this threshold is a fraction, where the numerator is:

$$-11 - 160\psi^2(1-\lambda)^2 - 2[37 - 2\psi(1-\lambda)(129 + 4\psi(1-\lambda)(39 - 251\psi(1-\lambda))) + 6\sqrt{75 - 3\psi(1-\lambda)(123 - \psi(1-\lambda)(-57 - 4\psi(1-\lambda)(-355 - 2\psi(1-\lambda)(-251 - 2\psi(1-\lambda)(14 + 37\psi(1-\lambda))))))}]^{\frac{1}{3}}$$
$$+ [37 - 2\psi(1-\lambda)(129 + 4\psi(1-\lambda)(39 - 251\psi(1-\lambda)) + 6\sqrt{75 - 3\psi(1-\lambda)(123 - \psi(1-\lambda)(-57 - 4\psi(1-\lambda)(-355 - 2\psi(1-\lambda)(-251 - 2\psi(1-\lambda)(14 + 37\psi(1-\lambda))))))}]^{\frac{2}{3}}$$
$$+ 16\psi(1-\lambda)(-1 + (37 - 2\psi(1-\lambda)(129 + 4\psi(1-\lambda)(39 - 251\psi(1-\lambda))) + 6\sqrt{75 - 3\psi(1-\lambda)(123 - \psi(1-\lambda)(-57 - 4\psi(1-\lambda)(-355 - 2\psi(1-\lambda)(-251 - 2\psi(1-\lambda)(14 + 37\psi(1-\lambda))))))}]^{\frac{1}{3}}))]$$

and the denominator is:

$$[3(37 - 2\psi(1-\lambda)(129 + 4\psi(1-\lambda)(-39 + 251\psi(1-\lambda)))) + 6\sqrt{75 - 3\psi(1-\lambda)(123 - \psi(1-\lambda)(-57 - 4\psi(1-\lambda)(-355 - 2\psi(1-\lambda)(-251 - 2\psi(1-\lambda)(14 + 37\psi(1-\lambda))))))}]^{\frac{1}{3}}$$

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